Probability theory
Lesson 21
The Binomial Distribution Table

21.1 - Using the Binomial Distribution Table.

Problem 1.1:
Since p = 0.50, go to the column shaded

p = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50

Since x = 15, we go along the left column to the value x = 15. From this value we connect to

p = 0.50.

p = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50

x
.
.
.
.

x = 15	0	0	0	0	0	0	0.0003	0.0016	0.0064	0.0207

Therefore, from the table, $P\{X \geq 15\} = 0.0207$.

21.1 - Problem 2:
Since p = 0.10 go to the column shaded

p = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50

Since x = 2, we go along the left column to the value x = 2. From this value we connect to

p = 0.50

p =	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50

x
.
.
.
.

x = 2	0.2642	0.6083	0.8244	0.9308	0.9757	0.9924	0.9979	0.9995	0.9999	1

Therefore, from the table, $P\{X \geq 2\} = 0.6083$.

21.1 - Problem 3:
It is not possible to find $P\{1 \leq X \leq 10\}$ directly from the table. However, the event

$\{X \geq 1\} = \{1 \leq X \leq 10\} \cup \{X \geq 11\}$.

Therefore,

$P\{X \geq 1\} = P\{1 \leq X \leq 10\} + P\{X \geq 11\}$

and it follows

$P\{1 \leq X \leq 10\} = P\{X \geq 1\} - P\{X \geq 11\} = 0.9885 - 0.0006 = 0.9879$.

21.1 - Problem 4:
It is not possible to find $P\{X = 15\}$ directly from the table.

However, the event $\{X \geq 15\} = \{X = 15\} \cup \{X \geq 16\}$. Therefore,

$P\{X \geq 15\} = P\{X = 15\} + P\{X \geq 16\}$

and it follows

$P\{X = 15\} = P\{X \geq 15\} - P\{X \geq 16\} = 0.0016 - 0.0003 = 0.0013$.

21.1 - Problem 5:
It is not possible to find $P\{X \leq 4\}$ directly from the table.

However,

$P\{X \leq 10\} = 1 - P\{X \geq 11\} = 1 - 0 = 1$.

21.2 - Real Life Applications.

21.2 - Problem 1:

➤(a).

We define success as the event that a report is a false alarm. Therefore, $p = 0.05$. Since the number of reports of fire is $n = 20$, we can use the binomial table. The event at least 2 of these 20 reports were false gives $\{X \geq 2\}$ and from the table

$P\{X \geq 5\} = .0.2642$.

where the column is $p = 0.05$ and the row is $x = 2$.

➤(b).

Step 1: We write the event "exactly 5 were false alarms" as $\{X = 5\}$.

Step 2: $\{X \geq 5\} = \{X = 5\} \cup \{X \geq 6\}$

Step 3: $P\{X \geq 5\} = P\{X = 5\} + P\{X \geq 6\}$

Step 4: $P\{X = 5\} = P\{X \geq 5\} - P\{X \geq 6\}$

From the table

$P\{X = 5\} = P\{X \geq 5\} - P\{X \geq 6\} = 0.0026 - 0.0003 = 0.0023$.

➤(c).

Step 1: We write the event "less than 5 are false alarms" as $\{X < 5\}$.

Step 2: $\{X < 5\} = \{X \geq 5\}'$

Step 3: $P\{X < 5\} = P\{X \geq 5\}' = 1 - P\{X \geq 5\}$

Step 4: From the table

$P\{X < 5\} = 1 - P\{X \geq 5\} = 1 - 0.0026 = 0.9974$.

21.2 Problem 2:

➤(a).

X represents the number of students that earn a grade of B or better.

Step 1: The event, less than 12 of these students will receive a grade of B or better can be written

$\{X < 12\} = \{X \geq 12\}'$.

Step 2: $P\{X < 12\} = P\{X \geq 12\}' = 1 - P\{X \geq 12\}$

Step 3: Using x = 12 and p = 0.25 in the table,

$P\{X < 12\} = P\{X \geq 12\}' = 1 - P\{X \geq 12\} = 1 - 0.0009 = .9991$.

➤(b).
Let X equal the number of students that earn a grade less than a B.

Step 1: For X we use p = 1 - 0.25 = 0.75.

Step 2: The event that less than half earn a grade less than a B is

$P\{X \leq 10\} = P\{X \geq 11\}' = 1 - P\{X \geq 11\}$.

Step 3: Using x = 11 and p = 0.75, the table give

$P\{X \leq 10\} = 1 - P\{X \geq 11\} = 1 - 0.9861 = 0.0039$.

➤(c).
Let X equal the number of students that earn a grade less than a B.

Step 1: For X, we use

p = 1 - 0.25 = 0.75.

Step 2: The event that more than 12 students earn a grade less than a B can be written

$\{X > 12\} = \{X \geq 13\}$.

Step 3: $P\{X > 12\} = P\{X \geq 13\} = 0.8982$

➤(d).
Let X equal the number of students that earn a grade B or better.

Step 1: The event that between 13 and 16 students earn a grade of B or better can be written

$\{13 \leq X \leq 16\}$.

Step 2: $\{13 \le X \le 16\} \cup \{X \ge 17\} = \{X \ge 13\}$

Step 3: $P\{13 \le X \le 16\} + P\{X \ge 17\} = P\{X \ge 13\}$

Step 4: For x = 13, 17 and p = 0.25, from the table we have

$P\{13 \le X \le 16\} = P\{X \ge 13\} - P\{X \ge 17\} = = 0.0002 - 0 = 0.0002.$

21.2 - Problem 3:
We first must find the probability that the student will pass a test by randomly guessing the correct answer. Let X equal the number of questions answered correctly.

Step 1: The probability that a question will be answered correctly is

p = 1/4 = 0.25.

Step 2: The event to get a c or better on a test is

$\{X \ge 10\}$.

Step 3: For x = 10 and p = 0.25, we have from the table

$P\{X \ge 10\} = 0.0139.$

Step 4: We next solve the binomial problem of the student passing only one test where

p = 0.0139, x = 1, N = 5 using the binomial formula

$$P\{Y = 1\} = \binom{5}{1}(0.0139)\,(0.9861)^4,$$

where Y equals the number of tests passed.

Step 5: $P\{Y = 1\} = \binom{5}{1}(0.0139)\,(0.9861)^4 \approx 0.066$

Supplementary Problems

1.
For each row in this table, we use the formulas

$P\{X \le 20\} = 1$ for $0 \le p \le 1$ and

$P\{X \le x\} = 1 - P\{X \ge x + 1\}$ for $0 \le x \le 20$.

$x = 0$: $P\{X \le 0\} = 1 - P\{X \ge 0 + 1\} = 1 - P\{X \ge 1\}$

$x = 1$: $P\{X \le 1\} = 1 - P\{X \ge 1 + 1\} = 1 - P\{X \ge 2\}$

$x = 2$: $P\{X \le 2\} = 1 - P\{X \ge 2 + 1\} = 1 - P\{X \ge 3\}$

$x = 3$: $P\{X \le 3\} = 1 - P\{X \ge 3 + 1\} = 1 - P\{X \ge 4\}$

$x = 4$: $P\{X \le 4\} = 1 - P\{X \ge 4 + 1\} = 1 - P\{X \ge 5\}$

$x = 5$: $P\{X \le 5\} = 1 - P\{X \ge 5 + 1\} = 1 - P\{X \ge 6\}$

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$x = 19$: $P\{X \le 19\} = 1 - P\{X \ge 19 + 1\} = 1 - P\{X \ge 20\}$

$x = 20$: $P\{X \le 20\} = 1 - P\{X \ge 20 + 1\} = 1$

This give the cumulative binomial distribution:

$P\{X \le x\}$ for N = 20 and

p = 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50.

For example, for x = 4

$P\{X \le 4\} = 1 - P\{X \ge 4 + 1\} = 1 - P\{X \ge 5\}$

1 - 0.0026 = 0.9974, 1 - 0.0432 = 0.9568, 1 - 0.1702 = 0.8298,

1 - 0.3704 = 0.6296, 1 -0.5852 = 0.4148, 1 - 0.7625 = 0.2375,

1 - 0.8818 = 0.1182, 1 - 0.9490 = 0.0510, 1 - 0.9811 = 0.0189

1 - 0.9941 = 0.0059.

p =	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
x = 4	.9974	.9568	.8298	.6296	.4148	.2375	.1182	.0510	.0189	.0059

Completing the rows gives the table P{X ≤ x}:

p =	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
x										
0	.3585	.1216	.0388	.0115	.0032	.0008	.0002	.0000	.0000	.0000
1	.7358	.3917	.1756	.0692	.0243	.0076	.0021	.0005	.0001	.0000
2	.9245	.6769	.4049	.2061	.0913	.0355	.0121	.0036	.0009	.0002
3	.9841	.8669	.6477	.4114	.2252	.1071	.0444	.0160	.0049	.0013
4	.9974	.9568	.8298	.6296	.4148	.2375	.1182	.0510	.0189	.0059
5	.9997	.9887	.9327	.8042	.6172	.4164	.2454	.1256	.0553	.0207
6	1	.9976	.9781	.9133	.7858	.6080	.4166	.2500	.1299	.0577
7	1	.9996	.9941	.9679	.8982	.7723	.6010	.4159	.2520	.1316
8	1	.9999	.9987	.9900	.9591	.8867	.7624	.5956	.4143	.2517
9	1	1	.9998	.9974	.9861	.9520	.8782	.7553	.5914	.4119
10	1	1	1	.9992	.9961	.9829	.9468	.8725	.7507	.5881
11	1	1	1	.9999	.9991	.9949	.9804	.9435	.8692	.7483
12	1	1	1	1	.9998	.9987	.9940	.9790	.9420	.8684
13	1	1	1	1	1	.9997	.9985	.9935	.9786	.9423
14	1	1	1	1	1	1	.9997	.9984	.9936	.9793
15	1	1	1	1	1	1	1	.9997	.9985	.9941
16	1	1	1	1	1	1	1	1	.9997	.9987
17	1	1	1	1	1	1	1	1	1	.9998
18	1	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1

2.
For each row in this table, we use the formulas:

$P\{X \leq 20\} = 1$ for $0 \leq p \leq 1$ and

$P\{X = x\} = P\{X \geq x\} - P\{X \geq x + 1\}$ for $x = 0,1,2,3,4,5$.

$x = 0$: $P\{X = 0\} = P\{X \geq 0\} - P\{X \geq 1\}$

$x = 1$: $P\{X = 1\} = P\{X \geq 1\} - P\{X \geq 2\}$

$x = 2$: $P\{X = 2\} = P\{X \geq 2\} - P\{X \geq 3\}$

$x = 3$: $P\{X = 3\} = P\{X \geq 3\} - P\{X \geq 4\}$

$x = 4$: $P\{X = 4\} = P\{X \geq 4\} - P\{X \geq 5\}$

$x = 5$: $P\{X = 5\} = P\{X \geq 5\} - P\{X \geq 6\}$

For $p = 0.25$, we compute the following from the binomial table $P\{X \geq x\}$:

$x = 0$: $P\{X = 0\} = P\{X \geq 0\} - P\{X \geq 1\} = 1 - 0.9968 = 0.0032$
$x = 1$: $P\{X = 1\} = P\{X \geq 1\} - P\{X \geq 2\} = 0.9968 - 0.9757 = 0.211$
$x = 2$: $P\{X = 2\} = P\{X \geq 2\} - P\{X \geq 3\} = 0.9757 - 0.9087 = 0.670$
$x = 3$: $P\{X = 3\} = P\{X \geq 3\} - P\{X \geq 4\} = 0.9087 - 0.7748 = 0.1339$
$x = 4$: $P\{X = 4\} = P\{X \geq 4\} - P\{X \geq 5\} = 0.7748 - 0.5852 = 0.1896$
$x = 5$: $P\{X = 5\} = P\{X \geq 5\} - P\{X \geq 6\} = 0.5852 - 0.3828 = 0.2024$

Completing the table:

Binomial distribution: P{X=x} for N = 20 & x = 0,...5.

p =	**0.05**	**0.10**	**0.15**	**0.20**	**0.25**	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**
x										
0	.3585	.1216	.0388	.0115	.0032	.0008	.0002	.0000	.0000	.0000
1	.3770	.2701	.1368	.0577	.0211	.0068	.0019	.0005	.0001	.0000
2	.1887	.2852	.2293	.1369	.0670	.0279	.0100	.0031	.0008	.0002
3	.0596	.1900	.2428	.2053	.1339	.0716	.0323	.0124	.0040	.0011
4	.0133	.0899	.1821	.2182	.1896	.1304	.0738	.0350	.0140	.0047
5	.0023	.0319	.1029	.1746	.2024	.1789	.1272	.0746	.0364	.0148

3.

➤a.

For the binomial table, we use the formula

$P\{X = x\} = P\{X \geq x \} - P\{X \geq x + 1\}$, $p = 0.65$, $x = 10$.

From the table,

$P\{X = 10\} = P\{X \geq 10 \} - P\{X \geq 11\} = 0.9468 - 0.8782 = 0.0686$.

➤b.

The event that at most 11 Navy personnel are over 70" tall can be written

$P\{X \leq 11\} = 1 - P\{X \geq 12\}$

For the binomial table we use $p = 0.55$ and $x = 12$,

$P\{X \leq 11\} = 1 - P\{X \geq 12\} = 1 - 0.4143 = 0.5857$.

➤c.

The event that at least 9 army personnel are 70" tall or under can be written as $P\{Y \geq 9\}$ where Y equals the number of army personnel under 70" tall or under. For the binomial table we use $p = 1 - 0.65 = 0.35$ and $x = 9$.

From the binomial table we have

$P\{Y \geq 9\} = 0.2376$.

➤d.

Step 1: The probability the event that at most 9 navy personnel are over 70" tall is

$P\{X \leq 9\} = 1 - P\{X \geq 10\} = 1 - 0.7505 = 0.2495$.

Step 2: The probability the event that at most 8 army personnel are over 70" tall is

$P\{X \leq 8\} = 1 - P\{X \geq 9\} = 1 - 0.9804 = 0.0196$.

Step 3: The event that at most 9 navy and 8 army personnel are over 70" tall can be written

$\{X \leq 9\} \cap \{X \leq 8\}$.

Step 4: It is reasonable to assume these two events are independent and therefore,

$P[\{X \le 9\} \cap \{X \le 8\}] = P\{X \le 9\}P\{X \le 8\} = (0.2495)(0.0196) \approx 0.0049.$

➤e.

Step 1:The event that at most 9 Navy or 8 Army personnel are over 70" tall can be written as

$\{X \le 9\} \cup \{X \le 8\}.$

Step 2:

$P[\{X \le 9\} \cup \{X \le 8\}] = P\{X \le 9\} + P\{X \le 8\} - P[\{X \le 9\} \cap \{X \le 8\}] =$

$P[\{X \le 9\} \cup \{X \le 8\}] = P\{X \le 9\} + P\{X \le 8\} - P\{X \le 9\}P\{X \le 8\} \approx$

$0.2495 + 0.0196 - 0.0049 = 0.2642$

4.

Since X is equal to the number of successes, the event $\{X \le 7\}$ is the event that at most 7 successes occurred Since N = 15 trials, we have X + Y = 15. Therefore,

$\{Y \le 7\} = \{X \ge 8\}, P\{Y \le 7\} = P\{X \ge 8\} = 1 - P\{X \le 7\} = 1 - 0.33 = 0.67$

5.

Step 1: From the binomial table for p = 0.50,

$P\{X = 12\} = P\{X \ge 12\} - P\{X \ge 13\} = 0.2517 - 0.1316 = 0.1201.$

Step 2: From the binomial table for p = 0.35,

$P\{Y = 10\} = P\{X \ge 11\} - P\{X \ge 10\} = 0.1218 - 0.0532 = 0.0682$

Step 3: $P\{X = 12; Y = 10\} = P\{X = 12\}P\{Y = 10\} = (0.1201)(0.0682) \approx 0.0082$

6.

Step 1: The closest values of p = 0.47 are $p_1 = 0.45$ and $p_2 = 0.50$

Step 2: From the binomial table $y_1 = 0.4086$ for $p_1 = 0.45$ and $y_2 = 0.5881$.

Step 3: For p = 0.47, the associated y value is given by

$$y = 0.4086 + \left(\frac{0.5881 - 0.4086}{0.50 - 0.45}\right)(0.47 - 0.45) = 0.4804.$$

7. From Table A, for $p = 0.45$,

$P\{X \geq 5 | X \leq 10\} = P\{(X \geq 5) \cap (X \leq 10)\}/P(X \leq 10)$

$P(X \leq 10) = 1 - P(X \geq 11) = 1 - 0.2493 = 0.7507$

$P\{(X \geq 5) \cap (X \leq 10)\} = P(5 \leq X \leq 10) = P(5 \leq X) - P(11 \leq X) = 0.9811 - 0.2493 = 0.7318$

$P\{X \geq 5 | X \leq 10\} = P\{(X \geq 5) \cap (X \leq 10)\}/P(X \leq 10) = 0.7318/0.7507 \approx 0.975$