Probability theory
Lesson 19
The Binomial Random Variable

19.1 - What is a Binomial Random Variable?

19.1 - Problem 1:
The sample space is

S = {(f,f,f), (21,f,f), (f,21,f),(f,f,21),(21,21,f),(21,f,21),(f,21,21),(21,21,21)}

where f means that the number 21 did not occur. For example (f,21,21) means that the first drawing did not result in a sum of 21 but the second and third drawings did. The random variable X will assign to each single outcome of **S** the number of drawings resulting in a sum of 21. For example X(f,21,21) = 2 since two of the drawings resulted a sum of 21.

Therefore,

Sample Space	Random Variable X
(f,f,f)	X(f,f,f) = 0
(21,f,f)	X(21,f,f) = 1
(f,21,f)	X(f,21,f) = 1
(f,f,21)	X(f,f,21) = 1
(21,21,f)	X(21,21,f) = 2
(21,f,21)	X(21,f,21) = 2
(f,21,21)	X(f,21,21) = 2
(21,21,21)	X(21,21,21) = 3

19.1 - Problem 2:
The sample space is **S** = {(f,f,f), (m,f,f), (f,m,f),(f,f,m),(m,m,f),(m,f,m),(f,m,m),(m,m,m)}.
where m means that a mystery was selected and f means that a mystery was not selected. For example (m,f,m) means that the first book drawn was a mystery, the second book drawn was not a mystery and the third book drawn was a mystery. The random variable will assign to peace single outcome in **S** the number of mystery novels selected. For example X(m,m,f) = 2, means that the first tow novels selected are mystery novels and the third novel is not a mystery novel.

Therefore,

Sample Space	Random Variable X
(f,f,f)	X(f,f,f) = 0
(m,f,f)	X(m,f,f) = 1
(f,m,f)	X(f,m,f) = 1
(f,f,m)	X(f,f,m) = 1
(m,m,f)	X(m,m,f) = 2
(m,f,m)	X(m,f,m) = 2
(f,m,m)	X(f,m,m) = 2
(m,m,m)	X(m,m,m) = 3

19.1 - Problem 3:
We define success (s) if the student has a strep throat and failure (f) if the student does not have a strep throat. For example (s,f,s,f) means the first and third students have strep throats and the second and fourth students do not have strep throats. The sample space is

S = {(s,s,s,f),(s,s,f,s),(s,f,s,s),(f,s,s,s),(,f,s,f),(s,s,f,f),(f,f,s,s),(f,s,f,s),(f,s,s,f),(s,f,f,s),(f,f,f,s),(f,f,s,f), (f,s,f,f),(s,f,f,f),(f,f,f,f),(s,s,s,s)}.

For each element of the sample space **S**, let X equal the number of students that have strep throats.

We have the following table:

Sample Space	Random Variable X
(s,s,s,s)	X(s,s,s,s) = 4
(s,s,s,f)	X(s,s,s,f) = 3
(s,s,f,s)	X(s,s,f,s) = 3
(s,f,s,s)	X(s,f,s,s) = 3
(f,s,s,s)	X(f,s,s,s) = 3
(s,f,s,f)	X(s,f,s,f) = 2
(s,s,f,f)	X(s,s,f,f) = 2
(f,f,s,s)	X(f,f,s,s) = 2
(f,s,f,s)	X(f,s,f,s) = 2
(f,s,s,f)	X(f,s,s,f) = 2
(s,f,f,s)	X(s,f,f,s) = 2
(f,f,f,s)	X(f,f,f,s) = 1
(f,f,s,f)	X(f,f,s,f) = 1
(f,s,f,f)	X(f,s,f,f) = 1
(s,f,f,f)	X(s,f,f,f) = 1
(f,f,f,f)	X(f,f,f,f) = 0

Supplementary Problems

1.

Step 1: The event $\{X > 1\} = \{X \geq 2\}$, two or more heads occurred when tossing a coin four times.

Step 2: From the sample space, we have $\{X \geq 2\} = \{X = 2\} \cup \{X = 3\} \cup \{X = 4\}$.

Step 3: $\{X = 2\}$ = (h,h,t,t),(h,t,h,t),(h,t,t,h),(t,t,h,h),(t,h,t,h),(t,h,h,t)

$\{X = 3\}$ = {(h,h,h,t),(h,h,t,h),(h,t,h,h),(t,h,h,h)}

$\{X = 4\}$ = {(h,h,h,h)}

Step 4: $\{X \geq 2\} = \{X = 2\} \cup \{X = 3\} \cup \{X = 4\}$ =

{(h,h,t,t),(h,t,h,t),(h,t,t,h),(t,t,h,h),(t,h,t,h),(t,h,h,t), (h,h,h,t),(h,h,t,h),(h,t,h,h),(t,h,h,h),(h,h,h,h)}

2.

Step 1: The event **E** = $\{2 \leq X \leq 4\}$ means that between 2 and 4 kings were drawn.
Let k stand for a king drawn and f for a king is not drawn. Then we have

Step 2: **E** = $\{2 \leq X \leq 4\} = \{X = 2\} \cup \{X = 3\} \cup \{X = 4\}$.

Step 3:

$\{X = 2\}$ =
{(k,k,f,f,f), (k,f,f,f,k), (k,f,k,f,f), (f,k,f,k,f), (f,f,k,k,f), (f,k,k,f,f), (k,f,f,k,f), (f,f,f,k,k), (f,f,k,f,k), (f,k,f,f,k)}

$\{X = 3\}$ =
{(f,f,k,k,k), (f,k,k,k,f), (f,k,f,k,k), (k,f,k,f,k), (k,k,f,f,k), (k,f,f,k,k), (f,k,k,f,k), (k,k,k,f,f), (k,k,f,k,f), (k,f,k,k,f)}

$\{X = 4\}$ =
{(f,k,k,k,k), (k,f,k,k,k), (k,k,f,k,k), (k,k,k,f,k), (k,k,k,k,f)}

Step 4: Therefore, **E** = $\{2 \leq X \leq 4\} = \{X = 2\} \cup \{X = 3\} \cup \{X = 4\}$ =

{(k,k,f,f,f), (k,f,f,f,k), (k,f,k,f,f), (f,k,f,k,f), (f,f,k,k,f), (f,k,k,f,f), (k,f,f,k,f), (f,f,f,k,k), (f,f,k,f,k), (f,k,f,f,k), (f,f,k,k,k), (f,k,k,k,f), (f,k,f,k,k), (k,f,k,f,k), (k,k,f,f,k), (k,f,f,k,k), (f,k,k,f,k), (k,k,k,f,f), (k,k,f,k,f),(k,f,k,k,f),(f,k,k,k,k), (k,f,k,k,k), (k,k,f,k,k), (k,k,k,f,k), (k,k,k,k,f)}

3.
Step 1: $\mathbf{E} = \{9 \leq X\} = \{X = 9\} \cup \{X = 10\}$

Step 2:
$\{X = 9\} =$
{(w,w,w,w,w,w,w,w,w,f), (w,w,w,w,w,w,w,w,f,w), (w,w,w,w,w,w,w,f,w,w),
(w,w,w,w,w,w,f,w,w,w), (w,w,w,w,w,f,w,w,w,w), (w,w,w,w,f,w,w,w,w,w),
(w,w,w,f,w,w,w,w,w,w), (w,w,f,w,w,w,w,w,w,w), (w,f,w,w,w,w,w,w,w,w),
(w,f,w,w,w,w,w,w,w,w), (f,w,w,w,w,w,w,w,w,w)}

$\{X = 10\}$ = {(w,w,w,w,w,w,w,w,w,w)}

Step 3: $\mathbf{E} = \{9 \leq X\} = \{X = 9\} \cup \{X = 10\} =$

{(w,w,w,w,w,w,w,w,w,f), (w,w,w,w,w,w,w,w,f,w), (w,w,w,w,w,w,w,f,w,w),
(w,w,w,w,w,w,f,w,w,w), (w,w,w,w,w,f,w,w,w,w), (w,w,w,w,f,w,w,w,w,w),
(w,w,w,f,w,w,w,w,w,w), (w,w,f,w,w,w,w,w,w,w), (w,f,w,w,w,w,w,w,w,w),
(f,w,w,w,w,w,w,w,w,w), (w,w,w,w,w,w,w,w,w,w)}

4.
$\mathbf{E_0} = \{X \leq 0\} = \{X = 0\}$ = {(t,t,t)}

$\mathbf{E_1} = \{X \leq 1\} = \{0 \leq X \leq 1\}$ = {(t,t,t),(t,t,h),(t,h,t),(h,t,t)}

$\mathbf{E_2} = \{X \leq 2\} = \{0 \leq X \leq 2\}$ = {(t,t,t),(t,t,h),(t,h,t),(h,t,t),(h,h,t),(h,t,h),(t,h,h)}

$\mathbf{E_3} = \{X \leq 3\} = \{0 \leq X \leq 3\}$ = {(t,t,t),(t,t,h),(t,h,t),(h,t,t),(h,h,t),(h,t,h),(t,h,h),(h,h,h)}

$\mathbf{F_0} = \{X \geq 0\} = \{0 \leq X \leq 3\}$ = {(t,t,t),(t,t,h),(t,h,t),(h,t,t),(h,h,t),(h,t,h),(t,h,h),(h,h,h)}

$\mathbf{F_1} = \{X \geq 1\} = \{1 \leq X \leq 3\}$ = {(t,t,h),(t,h,t),(h,t,t),(h,h,t),(h,t,h),(t,h,h),(h,h,h)}

$\mathbf{F_2} = \{X \geq 2\} = \{2 \leq X \leq 3\}$ = {(h,h,t),(h,t,h),(t,h,h),(h,h,h)}

$\mathbf{F_3} = \{X \geq 3\} = \{X = 3\}$ = {(h,h,h)}

5.
h: the single outcome of heads for each toss of the coin.
t: the single outcome of tails for each toss of the coin.
e: the singe outcome of even number for each toss of the die

o: the single outcome of an odd number for each toss of the die.

The sample space is **S =** {(h,e,e),(h,e,o),(h,o,e),(h,o,o),(t,e,e),(t,e,o),(t,o,e),(t,o,o)}

$\mathbf{E_0} = \{Z = 0\} = \{(t,o,o)\}$

$\mathbf{E_1} = \{Z = 1\} = \{(h,o,o),(t,e,o),(t,o,e)\}$

$\mathbf{E_2} = \{Z = 2\} = \{(h,e,o),(h,o,e),(t,e,e)\}$

$\mathbf{E_3} = \{Z = 3\} = \{(h,e,e)\}$

$\mathbf{F_0} = \{Z \le 0\} = \{Z = 0\} = \{(t,o,o)\}$

$\mathbf{F_1} = \{Z \le 1\} = \{0 \le Z \le 1\} = \{(t,o,o),(h,o,o),(t,e,o),(t,o,e)\}$

$\mathbf{F_2} = \{Z \le 2\} = \{0 \le Z \le 2\} = \{(t,o,o),(h,o,o),(t,e,o),(t,o,e),(h,e,o),(h,o,e),(t,e,e)\}$

$\mathbf{F_3} = \{Z \le 3\} = \{0 \le Z \le 3\} = \{(t,o,o),(h,o,o),(t,e,o),(t,o,e),(h,e,o),(h,o,e),(t,e,e),(h,e,e)\}$

6.
➤(a).
$\mathbf{E}_0 = \{(f,f,f)\}$

$\mathbf{E}_1 = \{(y,f,f), (f,y,f), (f,f,y)\}$

$E_2 = \{(y,y,f), (y,f,y), (f,y,y)\}$

$\mathbf{E}_3 = \{(y,y,y)\}$

➤(b). $\mathbf{F}_k = \{X \le k\}$

$\mathbf{F_0} = \{X \le 0\} = \{X = 0\} = \mathbf{E_0} = \{(f,f,f)\}$

$\mathbf{F_1} = \{X \le 1\} = \{0 \le X \le 1\} = \mathbf{E_0} \cup \mathbf{E_1} =$

$\{(f,f,f)\} \cup \{(y,f,f),(f,y,f),(f,f,y)\} = \{(f,f,f),(y,f,f),(f,y,f),(f,f,y)\}$

$\mathbf{F_2} = \{X \le 2\} = \{0 \le X \le 2\} = \mathbf{E_0} \cup \mathbf{E_1} \cup \mathbf{E_2} =$

$\{(f,f,f)\} \cup \{(y,f,f),(f,y,f),(f,f,y)\} \cup \{(y,y,f),(y,f,y),(f,y,y)\} =$

$\{(f,f,f),(f,f,f),(f,f,f),(f,f,f),(y,y,f),(y,f,y),(f,y,y)\}$

$\mathbf{F_3} = \{X \le 3\} = \{0 \le X \le 3\} = \mathbf{E_0} \cup \mathbf{E_1} \cup \mathbf{E_2} \cup \mathbf{E_3} =$

$\{(f,f,f)\} \cup \{(y,f,f),(f,y,f),(f,f,y)\} \cup \{(y,y,f),(y,f,y),(f,y,y)\} \cup \{(y,y,y)\} =$

$(f,f,f),(y,f,f),(f,y,f),(f,f,y),(y,y,f),(y,f,y),(f,y,y),(y,y,y)\}$

7.
x: a game was won on Friday.
a: a game was lost on Friday.

y: a game was won on Saturday.
b: a game was lost on Saturday.

z: a game was won on Sunday.
c: a game was lost on Saturday.

$\mathbf{E_0} = \{T \le 0\} = \{T = 0\} = \{(a_1,a_2,b,c)\}$

$\mathbf{E_1} = \{T \le 1\} = \{0 \le T \le 1\} = \{T = 0\} \cup \{T = 1\} =$

$\{(a_1,a_2,b,c)\} \cup \{(x_1,a_2,b,c),(a_1,x_2,b,c),(a_1,a_2,y,c),(a_1,a_2,b,z)\} =$

$\{(a_1,a_2,b,c),(x_1,a_2,b,c),(a_1,x_2,b,c),(a_1,a_2,y,c),(a_1,a_2,b,z)\}$

$\mathbf{E_2} = \{T \le 2\} = \{0 \le T \le 2\} = \{T = 0\} \cup \{T = 1\} \cup \{T = 2\} =$

$\{(a_1,a_2,b,c),(x_1,a_2,b,c),(a_1,x_2,b,c),(a_1,a_2,y,c),(a_1,a_2,b,z)\} \cup$

$\{(x_1,x_2,b,c),(x_1,a_2,y,c),(x_1,a_2,b,z),(a_1,x_2,y,c),(a_1,x_2,b,z),(a_1,a_2,y,z)\} =$

$\{(a_1,a_2,b,c),(x_1,a_2,b,c),(a_1,x_2,b,c),(a_1,a_2,y,c),(a_1,a_2,b,z),$

$(x_1,x_2,b,c),(x_1,a_2,y,c),(x_1,a_2,b,z),(a_1,x_2,y,c),(a_1,x_2,b,z),(a_1,a_2,y,z)\}$

$\mathbf{E_3} = \{T \le 3\} = \{0 \le T \le 3\} = \{T = 0\} \cup \{T = 1\} \cup \{T = 2\} \cup \{T = 3\} =$

$\{T = 0\} \cup \{T = 1\} \cup \{T = 2\} \cup \{(x_1,x_2,y,c),(x_1,x_2,b,z),(x_1,a_2,y,z),(a_1,x_2,y,z)\} =$

$\{(a_1,a_2,b,c),(x_1,a_2,b,c),(a_1,x_2,b,c),(a_1,a_2,y,c),(a_1,a_2,b,z),(x_1,x_2,b,c),(x_1,a_2,y,c),(x_1,a_2,b,z),(a_1,x_2,y,c),$

$(a_1,x_2,b,z),(a_1,a_2,y,z),(x_1,x_2,y,c),(x_1,x_2,b,z),(x_1,a_2,y,z),(a_1,x_2,y,z)\}$

$\mathbf{E_4} = \{T \le 4\} = \{0 \le T \le 4\} = \{T = 0\} \cup \{T = 1\} \cup \{T = 2\} \cup \{T = 3\} \cup \{T = 4\} =$

$\{T = 0\} \cup \{T = 1\} \cup \{T = 2\} \cup \{T = 3\} \cup \{(x_1,x_2,y,z)\} =$

$\{(a_1,a_2,b,c),(x_1,a_2,b,c),(a_1,x_2,b,c),(a_1,a_2,y,c),(a_1,a_2,b,z),(x_1,x_2,b,c),(x_1,a_2,y,c),$
$(x_1,a_2,b,z),(a_1,x_2,y,c),(a_1,x_2,b,z),(a_1,a_2,y,z),(x_1,x_2,y,c),(x_1,x_2,b,z),(x_1,a_2,y,z),$
$(a_1,x_2,y,z),(x_1,x_2,y,z)\}$

8.

➤(a).

$\mathbf{E}_p \cap \mathbf{F}_q = \{X \le p\} \cap \{X \ge q\} = \{q \le X \le p\}$

Case 1: $p < q$.

Since p is smaller than q, we have $\mathbf{E}_p \cap \mathbf{F}_q = \{q \le X \le p\} = \phi$.

Case 2: $p > q$.

Here we have $\mathbf{E}_p \cap \mathbf{F}_q = \{q \le X \le p\}$.

Case 3: $p = q$.

Here we have $\mathbf{E}_p \cap \mathbf{F}_q = \{q \le X \le p\} = \{X = p\}$.

➤(b).

Step 1: $\mathbf{E}_k = \{X \le k\}$

$\mathbf{E}_{k-1} = \{X \le k - 1\}$

$\mathbf{E}_{k-1}' = \{X \le k - 1\}' = \{X \ge k\}$

Step 2: $\{X = k\} = \{X \le k\} \cap \{X \ge k\} = \mathbf{E}_k \cap \mathbf{E}_{k-1}'$

➤(c).

Step 1: $\mathbf{F}_k = \{X \ge k\}$

$\mathbf{F}_{k+1} = \{X \ge k + 1\}$

$\mathbf{F}_{k+1}' = \{X \ge k + 1\}' = \{X \le k\}$

Step 2: $\{X = k\} = \{X \ge k\} \cap \{X \le k\} = \mathbf{F}_k \cap \mathbf{F}_{k+1}'$

➤(d).

$\mathbf{E}_k = \{X \le k\} = \{X \ge k + 1\}' = (\mathbf{F}_{k+1})'$