Statistical Inference Theory
Lesson 41
The F Distribution

41.1 - Applications

41.1 - Problem 1:

➤(a).

H_o: $\sigma_1^2 = \sigma_2^2$

H_a : $\sigma_2^2 > \sigma_1^2$

➤(b).

N_1 : 100 sampled from machine 1.

N_2 : 100 sampled from machine 2.

σ_1^2: variance measurement from machine 1.

σ_2^2 : variance measurement from machine 2.

s_1^2: .21, sample variance from machine 1.

s_2^2 : 0.33, sample variance from machine 2. .

From the null hypothesis, we assume $\sigma_1^2 = \sigma_2^2$. Therefore the equation can be written as

$$F = \frac{N_2(N_1 - 1)s_2^2\sigma^2{}_1}{N_1(N_2 - 1)s_1^2\sigma^2{}_1} = \frac{100(100 - 1)0.33}{100(100 - 1)0.21} \approx 1.57 \ .$$

➤(c).

Since $N_1 = N_2 = 100$,

$d_1 = N_1 - 1 = 100 - 1 = 99$ degrees of freedom.

$d_2 = N_2 - 1 = 100 - 1 = 99$ degrees of freedom.

Use the F distribution table E for $\alpha = 0.05$. Since the degrees of freedom for both samples is 99, we find the value $F_{0.05} = 1$.Since $F = 1.57 > 1$, we reject Ho and accept the conclusion that the machines are not functioning properly.

Using a level of significance of $\alpha = 0.05$ and $F = 2.15 > 1.84$ we reject H_0 : we accept the claim that the variation in grades for these females is greater than the male students.

➤(d).

Since $N_1 = N_2 = 100$,

$d_1 = N_1 - 1 = 100 - 1 = 99$ degrees of freedom.

$d_2 = N_2 - 1 = 100 - 1 = 99$ degrees of freedom.

Use the F distribution table E for $\alpha = 0.01$. Since the degrees of freedom for both samples is 99, we find the value $F_{0.01} = 1$.Since $F = 1.57 > 1$, we reject Ho and accept the conclusion that the machines are not functioning properly.

Supplementary Problems

1.

➤a.

Let σ_1^2 : the variance of the factory's machine.

Let σ_2^2 : the variance of the salesperson's machine.

H_0: $\sigma_2^2 = \sigma_1^2/2$

H_a: $\sigma_2^2 > \sigma_1^2/2$

➤b.

$$F = \frac{N_2(N_1 - 1)s_2^2\sigma_1^2}{N_1(N_2 - 1)s_1^2\sigma_2^2} = \frac{N_2(N_1 - 1)s_2^2 2\sigma_1^2}{N_1(N_2 - 1)s_1^2\sigma_1^2} = \frac{100(99)(2)0.13}{100(99)0.21} = 1.24$$

➤c.

For both $\alpha = 0.05$ and 0.01, $F_\alpha = 1$ for $N_1 = N_2 = 100$. Since $F = 1.24 > 1$, reject the salesperson claim.

2.

➤a.

H_o: $\sigma_1^2 = \sigma_2^2$

H_a: $\sigma_1^2 < \sigma_2^2$

➤b. Find F.

Here, we assume $\sigma_1^2 = \sigma_2^2$ is true. Therefore,

$$F = \frac{N_2(N_1 - 1)s_2^2\sigma_1^2}{N_1(N_2 - 1)s_1^2\sigma_2^2} = \frac{100(99)0.033}{100(99)0.021} = 1.57.$$

➤c.

For $N_1 = N_2 = 100$, $F_{0.005} = 1$. And since $F = 1.57 > 1$, we conclude the lubricant has no effect on the variance.

3.

➤a.

$H_o: \sigma_1^2 = \sigma_2^2$

$H_a: \sigma_1^2 \neq \sigma_2^2$

➤b.

Here, we assume $\sigma_1^2 = \sigma_2^2$ is true. Therefore,

$$F = \frac{N_2(N_1 - 1)s_2^2\sigma_1^2}{N_1(N_2 - 1)s_1^2\sigma_2^2} = \frac{21(20)4.1}{21(20)3.5} \approx 1.17$$

Since we need to test only one side, we use $\alpha/2 = 0.10 = 0.05$. For $N_1 = N_2 = 21$, we have 20 degrees of freedom from table E, $F_{0.05} = 2.12$.

And since $F = 1.17 < 2.12$, we conclude we have no statistical basis for rejecting the agency's claim.

➤c.

$$F = \frac{N_2(N_1 - 1)s_2^2\sigma_1^2}{N_1(N_2 - 1)s_1^2\sigma_2^2} = \frac{21(20)(4.1)(4.8)}{21(20)(3.5)(2.1)} \approx 2.67$$

Since $F = 2.67 > 2.12$, we reject the agency's claim.