# Statistical Inference Theory
# Lesson 32
# Type I Errors

## 32.1 - Two-sided test

**32.1 - Problem 1:**

➤(a).

Since the sample average $\bar{x} = 110$ is outside the interval $115 \le \bar{x} \le 135$,

the decision rule requires that you reject $H_o$ and accept $H_a$.

➤(b).

Step 1: Since the population average $\mu = 131$, the claim $H_o$: $\mu = 125$ is false and the counter-claim $H_a$: $\mu \neq 125$ is true.

Step 2: Since the sample average $\bar{x} = 110$ is outside the interval $115 \le \bar{x} \le 135$, the decision rule requires that you reject $H_o$ and accept $H_a$.

Step 3: Since you are rejecting $H_o$, which is false and accepting $H_a$ which is true, no Type I error occurred.

➤(c).

Step 1: Since the population average $\mu = 10$, the claim $H_o$: $\mu = 10$ is true and the counter-claim $H_a$: $\mu \neq 10$ is false.

Step 2: Since the sample average $\bar{x} = 110$ is outside the interval $115 \le \bar{x} \le 135$, the decision rule requires that you reject $H_o$ and accept $H_a$.

Step 3: Since you are rejecting $H_o$, which is true ($\mu = 125$) and accepting $H_a$ ,which is false, a Type I error has occurred.

**32.1 - Problem 2**:

➤(a).

We wish to test if a significant change in handling luggage has occurred. Therefore, we test against that no change has occurred: $\mu = 25{,}500$.

A significant change can occur by an increase or decrease in handling using the modified system. Therefore,

$H_o$: $\mu = 25{,}500$

$H_a$. $\mu \neq 25{,}500$

➤(b).
A type I error occurs when $H_o$ is true, but $H_o$ is rejected and $H_a$ is accepted.

$\mu = 25{,}500$

$$\sigma_{\overline{X}} = \frac{5000}{\sqrt{100}} = 500.$$

$$z = \frac{26000 - 25500}{500} = 1$$

$$z = \frac{25000 - 25500}{500} = -1$$

$\alpha = 1 - P\{25{,}000 \le \overline{X} \le 26{,}000\} = P\{-1 \le Z \le 1\} = 1 - (0.3413 + 0.3413) = 0.3174$

➤(c).
Step 1: Since $\mu = 25{,}500$, $H_o$ is true.

Step 2: The decision rule is:*D.R. : If 25,000* $\le \overline{X} \le$ *26,000 then* ***reject*** $\boldsymbol{H_a}$*; otherwise* ***reject*** $\boldsymbol{H_o}$ ***and accept*** $\boldsymbol{H_a}$**.**

Step 3: Since $\overline{x} = 25{,}897$, the decision rule requires us to reject $H_a$. Since we are rejecting $H_a$, which is false, we are not making a Type I error.

➤(d).
We start with decision rule:

*If 25,500 - e** $\le \overline{X} \le$ *25,500 + e* then* ***reject*** $\boldsymbol{H_a}$*; otherwise* ***reject*** $\boldsymbol{H_o}$ ***and accept*** $\boldsymbol{H_a}$**.**

Next, we need to find e* so that $\alpha = 0.05$.

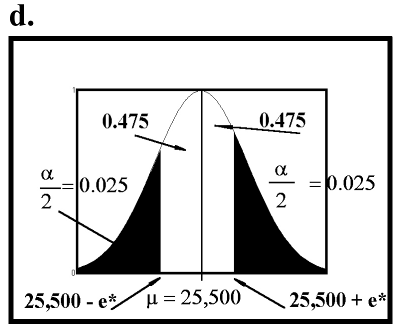
$e^* = z\sigma_{\overline{X}} = 500z.$

Using table C, we need to find z for $\alpha/2 = 0.025$.

$z = 1.96.$

Therefore, $e^* = 500(1.96) = 980$

**d.**



and

$25{,}500 - 980 \le \overline{X} \le 25{,}500 + 980 = 24{,}520 \le \overline{X} \le 26{,}480$

*D.R.*: *If* $24{,}520 \le \overline{X} \le 26{,}480$, *then* ***reject*** $H_a$***; otherwise reject*** $H_o$ ***and accept*** $H_a$***.***

**32.1-Problem 3:**

➤(a).

We test $\mu = 1{,}000$ electric sockets a minute against the counter- claim $\mu \neq 1{,}000$.

Therefore,

$H_o$: $\mu = 1{,}000$

$H_a$. $\mu \neq 1{,}000$

➤(b).

A type I error occurs when $H_o$ is true, but $H_o$ is rejected and $H_a$ is accepted.

$\mu = 1{,}000$

$$\sigma_{\overline{X}} = \frac{76}{\sqrt{36}} = 12.67.$$

$$z = \frac{1100 - 1000}{12.67} \approx 7.89$$

$$z = \frac{900 - 1000}{12.67} \approx -7.89$$

$\alpha = 1 - P\{900 \le \overline{X} \le 1100\} = P\{-7.89 \le Z \le 7.89\} = 1 - (0.4999 + 0.4999) \approx 0$

➤(c).

Step 1: Since $\mu = 1{,}000$, $H_o$ is true.

Step 2: The decision rule is:

*If* $900 \le \overline{X} \le 1100$ *then* ***reject*** $H_a$; *otherwise* **reject** $H_o$ **and accept** $H_a$**.**

Step 3: Since $\overline{x} = 1105$,

the decision rule requires us to reject $H_o$ and accept $H_a$. Since we are rejecting $H_o$ , which is true, we are making a Type I error.

➤(d).
We start with decision rule*: If 900 - e* ≤* $\overline{X}$ *≤ 1100 + e* then* ***reject*** $H_a$*; otherwise* ***reject*** $H_o$ ***and accept*** $H_a$.

Next, we need to find e* so that $\alpha = 0.01$.

$e^* = z\sigma_{\overline{X}} = 12.67z$.

Using table C, we need to find z for $\alpha/2 = 0.005$.

We look up z for the area 0.5 - 0.005 = 0.495 and find z = 2.58.

Therefore, e* = 12.67(2.58) ≈ 32.69 and

$1000 - 32.69 \le \overline{X} \le 1000 + 32.69 = 24{,}520 \le \overline{X} \le 26{,}480$

Therefore, we have the decision rule: *If 967.31 ≤* $\overline{X}$ *≤ 1032.69, then* ***reject*** $H_a$*; otherwise* ***reject*** $H_o$ ***and accept*** $H_a$.

# 32.2 - One-sided test

**32.2 - Problem 1:**
➤(a).
Step 1: The alternative to $H_a$ is $H_o$ .

Step 2: Since $H_a$: $\mu > 125$, the alternative to $\mu > 125$ is $\mu \le 125$.

Step 3: Since $H_o$ is the alternative to $H_a$, the other version for $H_o$ is $H_o$: $\mu \le 125$.

➤(b).
Since the sample average

$\bar{x} = 128.20$,

the decision rule requires that you **reject $H_o$ and accept $H_a$.**

➤(c).
Step 1: Since the population average $\mu = 139.71$, the claim $H_o$: $\mu \le 125$ is false and the counter-claim $H_a$: $\mu > 125$ is true.

Step 2: Since the sample average $\bar{x} = 128.20$, the decision rule requires that you **reject $H_o$ and accept $H_a$.**

Step 3: Since you are rejecting $H_o$, which is false and accepting $H_a$ which is true, no Type I error occurred.

➤(d).
Step 1: Since the population average $\mu = 100$, the claim $H_o$: $\mu \le 125$ is true and the counter-claim $H_a$: $\mu > 125$ is false.

Step 2: Since the sample average $\overline{x} = 128.20$, the decision rule requires that you **reject $H_o$ and accept $H_a$.**

Step 3: Since you are rejecting $H_o$, which is true and accepting $H_a$ ,which is false, a Type I error has occurred.

**32.2 - Problem 2:**
➤(a).
Since the claim is that the new laser machine will decrease the average production time, the claim and counter-claim is

$H_o$: $\mu = 5.6$

$H_a$: $\mu < 5.6$

➤(b).
A type I error occurs when $H_o$ is true, but $H_o$ is rejected and $H_a$ is accepted.

$H_o$: $\mu = 5.6$

$$\sigma_{\overline{X}} = \frac{0.45}{\sqrt{400}} \approx 0.0225$$

$$z = \frac{5 - 5.6}{0.0225} \approx -26.67$$

$\alpha = 0.5 - P\{\overline{X} \le 5\} = P\{Z \le -26.67\} = 0.5 - 0.4999 \approx 0$

➤(c).
Step 1: Since $\mu = 5.4$, $H_a$ is true.

Step 2: Since $\overline{x} = 5.2$,

the decision rule requires us to **reject $H_a$.**

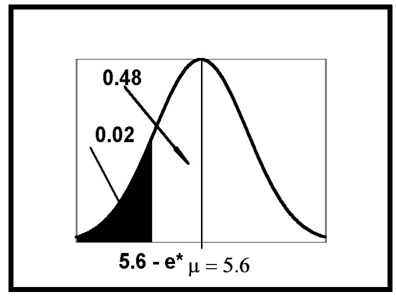
Step 3: Since we are rejecting $H_a$ which is true, no type I error occurs.

➤(d).
We start with the decision rule:

*If* $\overline{X} \leq 5.6 - e^*$ *then* ***reject*** $H_o$ ***and accept*** $H_a$***;*** *otherwise,* ***reject*** $H_a$***.***

Next, we need to find e* so that $\alpha = 0.02$.

**d.**




$e^* = z\sigma_{\overline{X}} = 0.0225z$.

Using table C, we need to find z for$\alpha = 0.02$.

We look up z for the area 0.5 - 0.02 = 0.48 and find z = 2.05.

Therefore, e* = 0.0225(2.05) ≈ 0.046 and

$\overline{X} \leq 5.6 - 0.046 = \overline{X} \leq 5.55$

Therefore, the decision rule is: *If* $\overline{X} < 5.55$*, then* ***reject*** $H_o$ ***and accept*** $H_a$*; otherwise,* ***reject*** $H_a$***.***

**32.2 - Problem 3:**
➤(a).
Since the claim is that no more than \$8.90 an hour is earned, we can set

$\mu \leq \$8.90$ or $\mu = \$8.90$.

Therefore,

$H_o$: $\mu = \$8.90$

$H_a$: $\mu > \$8.90$

➤(b).
A type I error occurs when $H_o$ is true, but $H_o$ is rejected and $H_a$ is accepted.

We assume $\mu = \$8.90$ is true.

$$\sigma_{\overline{X}} = \frac{1}{\sqrt{100}} = 0.1$$

$$z = \frac{9 - 8.9}{0.1} = 1$$

$\alpha = 0.5 - P\{\overline{X} > 9\} = P\{Z > 1\} = 0.5 - 0.3413 = 0.1587$

➤(c).
Step 1: Since $\mu = 8.79$, $H_o$ is true.

Step 2: Since $\bar{x} = \$9.79$, the decision rule requires us to reject $H_o$ and accept $H_a$.

Step 3: Since we are rejecting $H_o$ which is true, a Type I error has occurred.

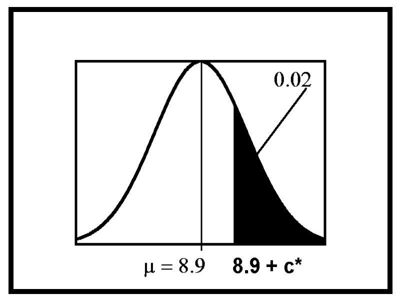
➤(d).
We start with the decision rule*: If* $\bar{X} > 8.9 + e^*$ *then* ***reject*** $H_o$ ***and accept*** $H_a$*; otherwise,* ***reject*** $H_a$.

Next, we need to find e* so that $\alpha = 0.01$.

$e^* = z\sigma_{\bar{X}} = (0.1)z.$

Using table C, we need to find z for $\alpha = 0.01$.

**d.**




We look up z for the area 0.5 - 0.01 = 0.49

and find $z = 2.33$.

Therefore, $e^* = (0.1)2.33$ and $\bar{X} > \$8.9 + 0.233 = 9.133$

Therefore, the decision rule is:

*If* $\bar{X} > \$9.133$*, then* ***reject*** $H_o$ ***and accept*** $H_a$*; otherwise,* ***reject*** $H_a$.

## Supplementary Problems

**1.**
➤a.
We assume the coin is well balance and therefore, p = 05 and $\mu = 0.5(100) = 50$.

If the coin is not well balanced then the coin can be biased for heads or for tails. Therefore,

$H_o$: $\mu = 50$

$H_a$: $\mu \neq 50$

➤b.
$\sigma = \sqrt{100(0.5)(0.5)} = 5$

$$z = \frac{55.5 - 50}{5} = 1.1$$

$$z = \frac{44.5 - 50}{5} = -1.1$$

From table C, $\alpha = P\{X < 45\} + P\{X > 55\} = P\{X < -1.1\} + P\{X > 1.1\} =$
$0.5 - 0.3643 + 0.5 - 0.3643 = 0.2714$

➤c.
*D.R.: If the number of heads is more than 50 + c* or less than 50 - c* heads,* ***reject the claim that the coin is well balanced.***

$c^* = z\sigma = 5z$

For $\alpha/2 = 0.025$, from table C, $z = 1.96$.

$c^* = 5(1.96) = 9.8$.

*D.R.: If the number of heads is more than 59 or less than 41 heads,* ***reject the claim that the coin is well balanced.***

**2.**
➤a.
The claim should be 60% of all medical students are female and the counter-claim is there are more than 60% or less than 60%. Therefore,

$H_o$: $\mu = 200(0.6) = 120$

$H_a$: $\mu \neq 120$

➤b.
$$\sigma = \sqrt{200(0.6)(0.4)} \approx 6.93$$

$$z = \frac{111.5 - 120}{6.93} = -1.23$$

From table C, $\alpha = P\{X < 112\} + P\{X >128\} = P\{X < -1.23\} + P\{X >-1.23\} =$
$0.5 - 0.3907 + 0.5 - 0.3907 = 0.2186$

➤c.
*D.R.: If in this sample, the number of females is more than 120 + c* or less than 120 - c* then* ***reject this claim.***

$c^* = z\sigma = 6.93z$

For $\alpha/2 = 0.005$, from table C, $z = 2.57$.

$c^* = 6.93(2.57) \approx 17.81$.

*D.R.: If in this sample, the number of females is more than 137 or less than 102 then* ***reject this claim***

**3.**

➤a.

The claim of the union is $p \leq 0.25$.

The counter-claim by the representative of the industry is$p > 0.25$.

Therefore,

$H_o: \mu \leq 200(0.25) = 50$ (union claim)

$H_a: \mu > 50$ ( industry counter-claim)

➤b.

$$\sigma = \sqrt{200(0.25)(0.75)} \approx 6.12$$

$$z = \frac{54.5 - 50}{6.12} = 0.74$$

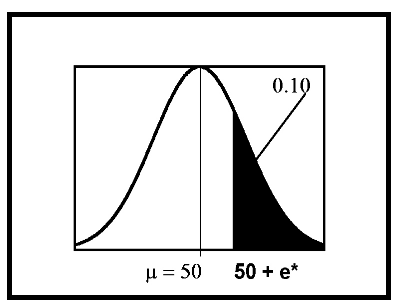
From table C, $\alpha = P\{X > 54\} = P\{Z > 0.74\} = 0.5 - 0.2704 = 0.2296$

➤c.

*DR: Let x represent the number of workers earning more than $12.00.*

*If x > 50 + e* then* ***reject the claim.***

**c.**




$e^* = z\sigma = 6.12z$

Since $\alpha = 0.10$,

we look up the area of

$0.5 - 0.10 = 0.4$

in table C, for $z = 1.28$.

Therefore $e^* = 7.83$

and the decision rule is written as *D.R.: Let x represent the number of workers earning more than $12.00. If x > 57 then* ***reject the claim.***

**4.**

➤a.

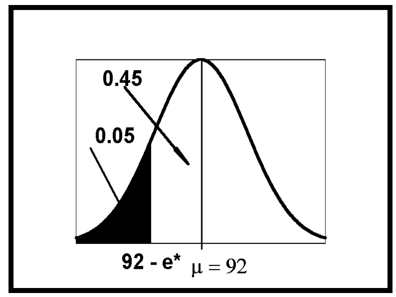
Since the manufacturer is biased in its claim, $p \geq 0.92$ and the counter-claim is $p < 0.92$.

➤b.

We start with the decision rule:

*D.R.: Let X represent the number of people that had a significant weight loss. If $X \leq c^*$, then* ***reject the claim ($H_o$) for*** $\alpha$ ***= 0.05.***

**b.**




Let $c^* = 92 - z\sigma$

$$\sigma = \sqrt{100(0.92)(0.08)} \approx 2.71$$

$c^* = 92 - 2.71z$

For $\alpha = 0.05$,

we look up the z value for the area

$0.5 - 0.05 = 0.45$.

From table C,

$z = 1.64$.

Therefore,

$c^* = 92 - 2.71(1.64) = 87.5$.

Therefore,

*D.R.: Let X represent the number of people that had a significant weight loss. If $X \leq 87$,* ***then reject the claim ($H_0$) for*** $\alpha$ ***= 0.05.***

**5.**

➤a.

We make the claim that the wheel is in balance. For this claim $p = 18/38$. Therefore,

$\mu = 76(18/38) = 36$.

$H_o$: $\mu = 36$

$H_a$: $\mu \neq 36$

➤b.
Let X represent the number of times an odd number occurred when spinning the wheel 76 times.

$$\sigma = \sqrt{76(\frac{18}{38})(\frac{20}{38})} \approx 4.35.$$

We need to find 1 - P{31.5 ≤ X ≤ 40.5}.

$$z = \frac{40.5 - 36}{4.35} \approx 1.03$$

$$z = \frac{31.5 - 36}{4.35} \approx -1.03$$

1 - P{31.5 ≤ X ≤ 40.5} = 1 - P{-1.03 ≤ X ≤ 1.03} = 1 - ( 0.3485 + 0.3485) = 1 - 0.697 = 0.303.

Therefore, α = 0.303.

➤c.
First, we state the decision rule as

*DR: If the number of odd numbers that occur, in the sample, is not between 36 - e* and 36 + e* then conclude that the wheel is not in balance.*

e* = 4.35z

Since α = 0.02,

we look up the area in table C, 0.5 - 0.02/2 = 0.49.

From table C, z = 2.33 and e* = 2.33(4.35) ≈10.14 .

Therefore,

*D.R.: If the number of odd numbers that occur, in the sample, is not between 26 and 46 then conclude that the wheel is not in balance.*

**6.**
➤a.
From the claim, p = 0.35. The counter-claim is p ≠ 0.35. Therefore,

$H_o$: μ = 0.35(500) = 175

$H_a$: μ ≠ 175 .

➤b.
We need to find P{169.5 ≤ X ≤ 180.5}.

$$\sigma = \sqrt{500(0.35)(0.65)} \approx 10.67$$

$$z = \frac{180.5 - 175}{10.67} \approx 0.52$$

$$z = \frac{169.5 - 175}{10.67} \approx -0.52$$

From table C, the area from the normal distribution is 0.1985.

P{169.5 ≤ X 180.5} = P{-0.52 ≤ Z ≤ 0.52} = 0.1985 = 0.1985 = 0.397

Therefore, $\alpha$ = 1 - 0.397 = 0.603.

➤c.
We state the decision rule as: *If, in this sample between 175 - c* and 175 + c* viewers are not males, then* ***reject the claim.***

c* = zσ = 10.67z

For the z value, we look up the area 0.5 - 0.03/2 = 0.485.

From table C, z = 2.17.

Therefore, c* = 10.67(2.17) ≈23.15.

The decision rule for $\alpha$ = 0.03: *If, in this sample between 152 and 198 viewers are not males, then* ***reject the claim****.*

**7.**
➤(a).
The claim can be interpreted as p ≥ 0.15.

The counter-claim is p < 0.15.

Therefore,

$H_o$: $\mu \geq 0.15(200) = 30$

$H_a$: $\mu < 30$

➤(b).
We need to find $\alpha$ = P{X ≤ 28.5}.

$\sigma = \sqrt{200(0.15)(0.85)} \approx 5.05$

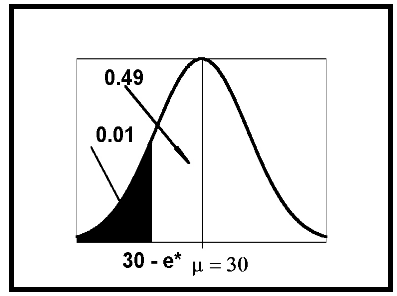
$z = \dfrac{28.5 - 30}{5.05} \approx -0.3$

For z = -0.3,

from table C, the area from the normal distribution is 0.1179.

$\alpha = P\{X \le 27.5\} = 0.5 - 0.1179 = 0.3821$

**c.**




➤(c).
We first write the decision rule as: *From the sample, let X represent the number of tire that lasted more than 35,000 miles.*

*If X ≤ 30 - c** ***then reject the claim: c* = z(5.05).***

For the area 0.5 - 0.01 = 0.49, z = 2.33. Therefore, c* = 2.33(5.05) = 11.67.

Therefore, the decision rule is

Therefore, the decision rule is: *From the sample, let X represent the number of tire that lasted more than 35,000 miles.*

*If X ≤ 30 - 11.67 ≈18 then* ***reject the claim.***

**8.**
➤a.
Since the CEO is biased, the claim can be written p ≤ 0.01 and therefore, the counter-claim is p > 0.01. Therefore,

$H_o: \mu = 0.01(1{,}000) = 10$ (or $\le 10$)

$H_a.: \mu > 10$

➤b.
To begin, we state the decision rule as follows: *If X ≥ e* + 10 then* ***reject $H_o$ and accept $H_a$****; otherwise* ***reject $H_a$.***

$c^* = z\sigma$

$\sigma = \sqrt{1000(0.01)(0.99)} \approx 3.15$

For $\alpha = 0.05$, we look up z for the area 0.5 - 0.05 = 0.45 in table C, $z = 1.64$.

$c^* = (1.64)(3.15) = 5.166$

$10 + 5.166 \approx 16$

The decision rule can be stated as

*D.R. $x \geq 16$ then reject* ***the claim that only 1% of the trucks have been recalled.***

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