Statistical Inference Theory
Lesson 31
Hypothesis Testing

31.1- Two sided-test

31.1 - Problem 1:
Since the airport is interested in testing if there is a significant change in the amount of luggage handled, the claim is that there has been no change:

➤(a).
H_o: $\mu = 25{,}500$

and the counter-claim is that there has been a significant change: H_a: $\mu \neq 25{,}500$

➤(b).

Step 1: First we compute $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{N}} = \dfrac{5000}{\sqrt{100}} = 500.$

Step 2: From the alternative hypothesis, we have a two-sided test.

Step 3: We need to compute $P\{25{,}000 \leq \overline{X} \leq 26{,}000\}$.

$$z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \frac{26{,}000 - 25{,}500}{500} = 1$$

$$z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \frac{25{,}000 - 25{,}500}{500} = -1$$

From the normal distribution table, we find $P\{-1 \leq Z \leq 1\} = 0.3413 + 0.3413 = 0.6826$.

and $P\{25{,}000 \leq \overline{X} \leq 26{,}000\} = P\{-1 \leq Z \leq 1\} = 0.6826$.

Step 4: Therefore, the probability of rejecting H_o and accepting H_a is $1 - 0.6826 = 0.3174$.

and the probability of rejecting H_a is 0.6826 .

➤(c).
Since, 24,900 < 25,000,

by the decision rule we reject H_0 and accept H_a. We conclude that this new method has changed the handling of passengers' luggage.

31.1 - Problem 2:
➤(a).
Since it is claimed the machine produces 1,000 electric sockets a minute, we have

H_o: $\mu = 1{,}000$
and the counter-claim is that there has been a significant change:

H_a: $\mu \neq 1{,}000$

➤(b).

Step 1: First we compute $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{N}} = \dfrac{76}{\sqrt{36}} = 12.67.$

Step 2: From the alternative hypothesis, we have a two-sided test.

Step 3: We need to compute $P\{900 \le \overline{X} \le 1100\}$.

$$z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \frac{1100 - 1000}{12.67} \approx 7.89$$

$$z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \frac{900 - 1000}{12.67} \approx -7.89$$

From the normal distribution table, we find $P\{-7.89 \le Z \le 7.89\} = 0.4999 + 0.4999 \approx 1$.

and $P\{900 \le \overline{X} \le 1100\} = P\{-7.89 \le Z \le 7.89\} \approx 1$.

Step 4: Therefore, the probability of rejecting H_o and accepting H_a is 1 - 1 = 0

and the probability of rejecting H_a is 1.

➤(c).
Since, $900 \le 1{,}097 \le 1100\}$,

the decision rule requires us to reject H_a.

31.2 - One sided-test

31.2 - Problem 2:

➤(a).

Since the claim is that the new laser machine will decrease the average production time, we use this as the counter-claim and have

H_a: $\mu < 5.6$

which will test

H_o: $\mu = 5.6$

➤(b).

Step 1: First we compute $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{N}} = \dfrac{0.45}{\sqrt{100}} = 0.045.$

Step 2: From the alternative hypothesis, we have a one-sided test.

Step 3: $z = \dfrac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \dfrac{5.5 - 5.6}{0.045} \approx -2.22$

From the normal distribution table, we find $P\{ Z < -2.22\} = 0.5 - 0.4868 = 0.0132$.

$P\{\overline{X} < 5.5\} = P\{ Z < -2.22\} = 0.0132$.

Step 4: Therefore, the probability of rejecting H_o and accepting H_a is 0.1132 .

and the probability of rejecting H_a is 1 - 0.0132 = 0.0.9868.

➤(c).

Since, $\overline{X} = 5.1 < 5.5$, reject H_o and accept H_a .

31.2 - Problem 2:

➤(a).

Since the claim is that the workers earn no more than \$8.90 an hour, we have H_a: $\mu \leq \$8.90$ which will test

H_o: $\mu > \$8.90$.

➤(b).

Step 1: First we compute $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \frac{1}{\sqrt{100}} = 0.1.$

Step 2: From the alternative hypothesis, we have a one-sided test.

Step 3: $z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \frac{9 - 8.9}{0.1} \approx 1$

From the normal distribution table, we find P{ Z > 1} = 0.5 - 0. = 0.3413 = 0.1587

$P\{\overline{X} > 9\} = P\{Z > 1\} = 0.1587$.

Step 4: Therefore, the probability of rejecting H_0 and accepting H_a is 0.11587

and the probability of rejecting H_a is 1 - 0.1587 = 0.8413.

➤(c).
Since, $\overline{X}$ = \$8.80 < \$9.00 then reject H_a. There is not statistical basis to reject the union's claim.

Supplementary Problems

1.
➤a .
The claim is that the coin is well balanced. Therefore, p = 0.5,

the probability that the coin tossed will result in heads on each toss and $\mu = 0.5(100) = 50$.

It follows H_o: $\mu = 50$.

The counter-claim is that the coin is not well balanced. Since the bias on the coin can be in favor of heads or tails, $p \neq 0.5$.

It follows H_a: $\mu \neq 50$

➤b.
Let X represent the number of heads that results from tossing the coin 100 times. The decision rule can be written out as

DR: If $45 \le X \le 55$, then reject H_a and accept H_o; otherwise reject H_o and accept H_a.

➤c.
Since H_o is true, $\mu = 50$, we reject H_a and accept H_0. X is a binomial random variable which can be approximated by the normal distribution. Therefore, $\sigma = \sqrt{(100)(0.5)(0.5)} = 5.$

$P\{45 \le X \le 55\}$ is the probability of rejecting H_a and accepting H_o.

$$z = \frac{55.5 - 50}{5} = 1.1$$

$$z = \frac{44.5 - 50}{5} = -1.1$$

From the normal distribution, $P\{45 \le X \le 55\} = P\{-1.1 \le Z \le 1.1\} = 0.3643 + 0.3643 = 0.7286$.

➤d.
The probability of rejecting H_o and accepting H_a when H_o is true is $1 - P\{45 \le X \le 55\} = 0.2714$.

2.
➤a.
Since the claim is that 60% of all medical students are female, p = 0.6.

Since, N = 200,

H_o: $\mu = 200(0.6) = 120$

The counter-claim is neutral and therefore, H_a: $\mu \ne 120$.

➤b.
Let X represent the number of students that are female in the sample. The decision rule can be written out as

DR: If $112 \le X \le 128$, then reject H_a and accept H_a; otherwise reject H_0 and accept H_a.

➤c.
We need to find $P\{112 \le X \le 128\}$, the probability that the claim is accepted (H_o) and H_a rejected when H_o is true.

Since, X is a binomial random variable,

$$\sigma = \sqrt{120(0.6)(0.4)} \approx 5.37$$

$$z = \frac{128.5 - 120}{5.37} \approx 1.58$$

$$z = \frac{111.5 - 120}{5.37} \approx -1.58 .$$

From the normal distribution table, $P\{112 \le X \le 128\} = P\{-1.58 \le Z \le 1.58\} = 0.4429 + 0.4429 = 0.8858$.

➤d.
The probability that the claim is rejected (H_o) and H_a accepted when H_o is true is

$1 - P\{112 \le X \le 128\} = 1 - 0.907 = 0.093$.

3.
➤a.
Since the claim is that p = 0.95,

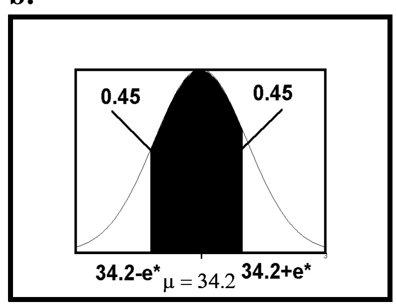
H_o: $\mu = 0.95(36) = 34.2$.

Since the machine can fill less than or more than 16 ounces, H_a: $\mu \ne 34.2$.

➤b.
We need to find e*, so that $P\{34.2 - e^* \le X \le 34.2 + e^*\} = 0.9$,

the probability that the claim is accepted (H_o) and H_a rejected when H_o is true.

b.



Since, X is a binomial random variable,

$$\sigma = \sqrt{36(0.95)(0.05)} \approx 1.31$$

$e^* = z\sigma = z(1.31)$

For the area 0.9/2 = 0.45, from the normal distribution table,

z = 1.64.

Therefore,

$e^* = 1.31(1.64) \approx 2.15$.

➤c.
DR: From the sample, let X represent the number of bottles that contain 16 ounces of water.

If 32. $\leq X \leq$ 36 then accept the claim H_o and reject H_a; otherwise reject the claim H_o and accept H_a.

4.

➤a.

On the wheel, there are 38 'numbers' and 18 odd numbers. If the wheel is in balance, then $p = 18/38$ and

H_o: $\mu = 76(18/38) = 36$.

If the wheel is out of balance then H_a: $\mu \neq 36$.

➤b.

On spinning the wheel 76 times, let X equal the number of odd numbers that occur. Therefore,

DR: If $32 \leq X \leq 40$, then reject H_a and accept H_0; otherwise reject H_o and accept H_a.

➤c.

We need to find $P\{32 \leq X \leq 40\}$, the probability that the claim is accepted (H_o) and H_a rejected when H_o is true.

Since, X is a binomial random variable, $\sigma = \sqrt{76(\frac{18}{38})(\frac{20}{38})} \approx 4.35$.

$$z = \frac{31.5 - 36}{4.35} \approx -1.03$$

$$z = \frac{40.5 - 36}{4.35} \approx 1.03$$

From the normal distribution table, $P\{32 \leq X \leq 40\} = P\{-1.03 \leq Z \leq 1.03\} = 0.3485 + 0.3485 = 0.697$.

➤d.

$1 - P\{32 \leq X \leq 40\} = 1 - 0.697 = 0.303$, the probability of rejecting H_o and H_a accepted when H_o is true.

5.

➤a.

Since $N = 500$, $p = 0.35$, $\mu = 500(0.35) = 175$.

Therefore, H_o: $\mu = 175$

and since the counter-claim is neutral, H_a: $\mu \neq 175$.

➤b.
Let X be a binomial random variable that equals the number of men that watch soap operas taken from the sample.

DR: If $170 \leq X \leq 180$, then reject H_a and accept H_0; otherwise reject H_o and accept H_a.

➤c.
Since H_o is true,

$\mu = 175$

$$\sigma = \sqrt{500(0.35)(0.65)} \approx 10.67$$

We need to find $P\{170 \leq X \leq 180\}$, which is equal to the probability that the claim is accepted (H_o) and H_a rejected when H_o is true.

$$z = \frac{180.5 - 175}{10.66} \approx 0.52$$

$$z = \frac{169.5 - 175.5}{10.66} \approx -0.52$$

$P\{170 \leq X \leq 180\} = P\{-0.52 \leq X \leq 0.52\} = 0.1985 + 0.1985 = 0.397$

➤d.
The probability that the claim is rejected (H_o) and H_a accepted when H_o is true is

$1 - P\{170 \leq X \leq 180\} = 1 - 0.397 = 0.603.$

6.
➤a.
We assume $p = 0.15$.

Then $\mu = 0.15(200) = 30$ and $\sigma = \sqrt{200(0.15)(0.85)} \approx 5.05$.

Therefore,

$H_o : \mu = 30$
$H_a : \mu \neq 30$

➤b.
e* = $z\sigma$ = 1.96(5.05) ≈ 9.9.

➤c.
Since, e ≈ 9.9, we rewrite the decision rule as:

DR: From the sample let X represent the number of tire that lasted more than 35,000 miles. If 20.9 ≤ X ≤ 39.9 then accept the assumption; otherwise reject the assumption.

7.
➤a.
Since we are testing if system and 100 games are played, H_o: $\mu = 0.60(100) = 60$and the counterclaim will be $H_a : \mu < 60$

➤b.
From the company claim, $p \geq 0.60$.

Therefore, $\mu = 0.6(100) = 60$ and $\sigma = \sqrt{100(0.6)(0.4)} \approx 4.9$.

Therefore,

$H_o : \mu \geq 60$
$H_a : \mu < 60$.

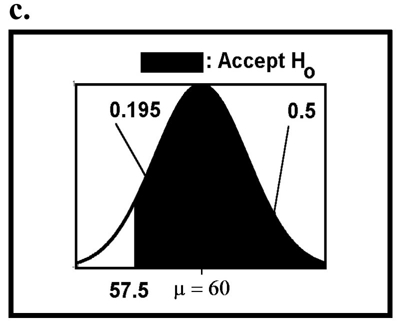
➤c.
From the decision rule, the probability that H_o is accepted equals P{X ≥57.5}.

$$z = \frac{57.5 - 60}{4.9} \approx -0.51$$

From the normal distribution table, P{X ≥ 57.5} = P{Z ≥ -0.51} = 0.195 + 0.5 = 0.6950

➤d.
1 - 0.695 = 0.305

c.



8.
➤a.
The claim is $p \leq 0.25$.

Therefore, $\mu \leq 50$ and $\sigma = \sqrt{200(0.25)(0.75)} \approx 6.12$.

Therefore,

$H_o: \mu \leq 50$
$H_a: \mu > 50.$

➤b.
From the decision rule, we need to find $P\{X \leq 45.5\}$.

$$z = \frac{45.5 - 50}{6.12} \approx -0.74$$

From the normal distribution table, $P\{X \leq 45.5\} = P\{Z \leq -0.74\} = 0.5 - 0.2704 = 0.2296$.

➤c.
$1 - P\{X \leq 45.5\} = 1 - 0.2296 = 0.7704$

➤d.
DR: Let X represent the number of workers earning more than \$12.00. If X ≤ 59 then accept the claim of the union. If X > 60 then reject their claim.