Statistical Inference Theory
Lesson 28
The CENTRA LIMIT THEOREM

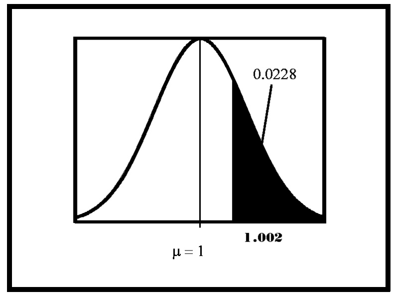
28.1 -What is the Central Limit Theorem for $\overline{x}$?

28.1 - Problem 1:

➤(a).

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{N}} = \frac{0.01}{\sqrt{100}} = 0.001$$

b.



➤(b).

We use the formula $z = \frac{\overline{X} - \mu}{\sigma_{\overline{x}}}$

to find the area under the normal distribution curve for $\overline{X} = 1.002$.

$\mu = 1$

$\sigma_{\overline{X}} = 0.001$

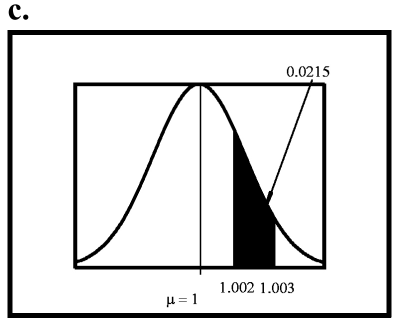
$$z = \frac{1.002 - 1}{0.001} = \frac{0.002}{0.001} = 2$$

From the normal distribution tables, $P\{\overline{X} \geq 1.002\} = P\{Z \geq 2\} = 0.5 - 0.4772 = 0.0228$

➤(c).

$$z = \frac{1.002 - 1}{0.001} = \frac{0.003}{0.001} = 3$$

c.



From the normal distribution table,

$P\{1.02 \leq \overline{X} \leq 1.003\} = P\{2 \leq z \leq 3\} =$

$0.4987 - 0.4772 = 0.0215.$

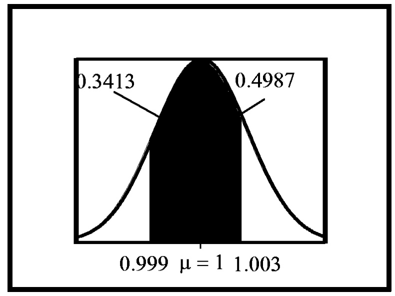
➤(d).

$$z = \frac{0.999 - 1}{0.001} = -\frac{0.001}{0.001} = -1$$

From the normal distribution table,

$P\{0.999 \le \overline{X} \le 1.003\} = P\{-1 \le Z \le 3\} = 0.3413 + 0.4987 = 0.84.$

d.



28.1 - Problem 2:

Step 1:To solve for μ, we use the formula $\mu = \overline{X} - z\sigma_{\overline{X}}$.

Step 2: $\overline{X} = 100$

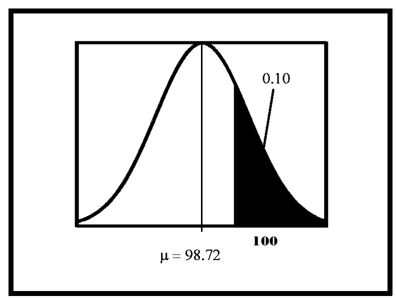
Step 3: $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{36}} = 1$

Step 4:We need to find z such that $P\{\overline{X} > 100\} = 0.10 = 0.5 - 0.40.$

The area 0.40 from the normal distribution table: gives z = 1.28.

Step 5: $\mu = \overline{X} - z\sigma_{\overline{X}} = 100 - (1.28)(1) \approx 98.72$ pounds.

2.



28.1 - Problem 3:

Step 1:To solve for c*, we use the formula

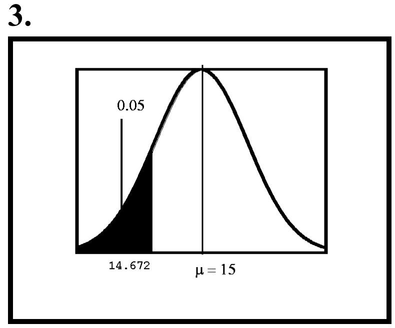
$c^* = \mu + z\sigma_{\overline{X}}$

Step 2: μ = 15

Step 3: $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1.4}{\sqrt{49}} = \frac{1.4}{7} = 0.2$

Step 4: We need to look up the z value for $P\{\overline{X} < c^*\} = 0.05 = 0.5 - 0.45$

3.



From the Normal distribution table: z = -1.64.

Step 5: $c^* = \mu + z\sigma_{\overline{X}} = 15 - 1.64(0.2) = 14.672$ inches.

c* = 14.67 inches.

Supplementary Problems

1.
μ = 178

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{100}} = \frac{\sigma}{10}$$

$$\mu = \overline{X} - z\sigma_{\overline{X}} = \overline{X} - z\frac{\sigma}{\sqrt{N}} = 179 - z\frac{\sigma}{\sqrt{100}} = 179 - z\frac{\sigma}{10} = 178$$

$$\sigma = \frac{10}{z}$$

$$P\{\overline{X} \geq 179\} = 0.40$$

From the normal distribution table for the area 0.50 - 0.40 = 0.10, $z \approx 0.25$.

$$\sigma = \frac{10}{z} = \frac{10}{0.25} = 40$$

2.

➤a.

$$0 + 1 + 2 + 3 + ... + N = 1 + 2 + 3 + ... + 100 = \frac{100(100 + 1)}{2} = 5050$$

$$\mu = \frac{5050}{101} = 50$$

➤b.

$$\sigma^2 = E(\overline{X}^2) - E(\overline{X})^2$$

$$E(\overline{X}^2) = \frac{0^2 + 1^1 + 2^2 + 3^2 + ... + 100^2}{101} = \frac{100(100 + 1)((2(100) + 1)}{606} = 3350$$

$$\sigma^2 = E(\overline{X}^2) - E(\overline{X})^2 = 3350 - 50^2 = 850.$$

➤c.

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}} = \frac{\sqrt{850}}{\sqrt{36}} \approx 4.86$$

$$z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \frac{60 - 50}{4.86} \approx 2.06$$

From the normal distribution table, for $z = 2.06$, the area is 0.4803 .

Therefore, $P\{\overline{X} > 60\} = 0.5 - 0.4803 \approx 0.02$.

➤d.

The sample space is **S** = {0,1,2,3,4,5, ... 100} and **E** = {X > 60}.

The $P(\mathbf{E}) = \frac{40}{101} \approx 0.40$

➤e.

$$\frac{\sigma}{\sqrt{N}} = \frac{\sqrt{850}}{\sqrt{N}} = 4.86$$

Solving the equation for N gives N ≈ 36.

➤f.

$P\{\overline{X} \geq 60\} = P\{Z \geq z\} = 0.01$

From the normal distribution table, z = 2.33 for the area 0.5 - 0.01 = 0.49.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{N}}} = \frac{\sqrt{N}[60 - 50]}{29.15} = 2.33$$

Solving for N, we get N ≈ 46.

3.

➤a.

For the female students we have

$$z = \frac{2.97 - 2.95}{\frac{0.2}{\sqrt{200}}} = 1.41 .$$

$P\{\overline{X} \geq 2.97\} = P\{Z \geq 1.41\} = 0.5 - 0.4207 = 0.0793$

For the male students we have

$$z = \frac{2.97 - 2.94}{\frac{0.25}{\sqrt{100}}} = 1.2$$

$P\{\overline{Y} \geq 2.97\} = P\{Z \geq 1.2\} = 0.5 - 0.3849 = 0.1151$

Since it is reasonable to assume independent of grade point averages between men and women, we multiply to get the probability that the sampled female and male students GPA is greater than 2.97: (0.0793)(0.1151) = 0.009 .

➤b.

Since we want the probability that the average G.P.A. of the sampled female students **OR** male students is greater than 2.97, we use the formula

P(**A**∪**B**) = P(**A**) + P(**B**) - P(**A**)P(**B**) = 0.0793 + 0.1151 - 0.009 = 0.1854.

4.
First we find the probability that the machine is stopped.

$$z = \frac{16 - 15.85}{\frac{0.7}{\sqrt{100}}} \approx -2.14$$

$P\{\overline{X} < 15.85\} = P\{Z < -2.14\} = 0.5 - 0.4838 \approx 0.02$

To find the probability that over a 5 hour period, the machine will be stopped 1 time is a binomial problem where

p = 0.02
N = 5,
k = 1.

$P\{X = 2\} = \binom{5}{1}(0.02)^1(0.98)^4 \approx 0.09$.

5.
➤a.
Step 1:

X_1 : 1,2,3,4,5,6

X_2 : 1,2,3,4,5,6

Since

$\overline{X}$ =

(1 + 1)/2 = 1, (1 + 2)/2 = 3/2, (1 + 3)/2 = 2, (1 + 4)/2 = 5/2, (1 + 5)/2 = 3, (1 + 6)/2 = 7/2,

(2 + 3)/2 = 5/2, (2 + 4)/2 = 3, (2 + 5)/2 = 7/2, (2 + 6)/2 = 4,

(3 + 4)/2 = 7/2, (3 + 5)/2 = 4, (3 + 6)/2 = 9/2,

(4 + 5)/2 = 9/2, (4 + 6)/2 = 5,

(5 + 6)/2 = 11/2,

we have the following table:

$\overline{\mathbf{X}} = \overline{\mathbf{x}}$	$P\{\overline{\mathbf{X}} = \overline{\mathbf{x}}\}$
2/2	1/36
3/2	2/36
4/2	3/36

5/2	4/36
6/2	5/36
7/2	6/36
8/2	5/36
9/2	4/36
10/2	3/36
11/2	2/36
12/2	1/36

➤b.

$E(X_1) = E(X_2) = (1 + 2 + 3 + 4 + 5 + 6)/6 = 3.5$

To compute $E(\overline{X})$, we complete the following table:

$\overline{\mathbf{X}} = \overline{\mathbf{x}}$	$P\{\overline{\mathbf{X}} = \overline{\mathbf{x}}\}$	$\overline{\mathbf{x}}P\{\overline{\mathbf{X}} = \overline{\mathbf{x}}\}$
2/2	1/36	2/72
3/2	2/36	6/72
4/2	3/36	12/72
5/2	4/36	20/72
6/2	5/36	30/72
7/2	6/36	42/72
8/2	5/36	40/72
9/2	4/36	36/72
10/2	3/36	30/72
11/2	2/36	22/72
12/2	1/36	12/72

$E(\overline{X})$ = 252/72 = 3.5

➤c.

Step 1: $\sigma^2 = E(X_1^2) - E(X_1)^2$

Step 2:

x^2	$P\{X^2 = x^2\}$	$x^2 P\{X^2 = x^2\}$
1	1/6	1/6
4	1/6	4/6
9	1/6	9/6
16	1/6	16/6
25	1/6	25/6
36	1/6	36/6
		$E(X_1^2) = 91/6$

$E(X_1)^2 = 7/2^2 = 49/4$.

$\sigma^2 = E(X_1^2) - E(X_1)^2 = 91/6 - 49/4 = 35/12$

Step 3:

$\overline{X}^2 = \overline{x}^2$	$P\{\overline{X}^2 = \overline{x}^2\}$	$\overline{x}^2 P\{\overline{X}^2 = \overline{x}^2\}$
4/4	1/36	4/144
9/4	2/36	18/144
16/4	3/36	48/144
25/4	4/36	100/144
36/4	5/36	180/144
49/4	6/36	294/144
64/4	5/36	320/144
81/4	4/36	324/144
100/4	3/36	300/144
121/4	2/36	242/144
144/4	1/36	144/144

$E(\overline{X}^2) = 1974/144$

Step 3: $\sigma^2_{\overline{X}} = E(\overline{X}^2) - E(\overline{X})^2 = 1974/144 - (7/2)^2 = 35/24$

➤d.

Step 1: $\sigma^2_{X_1+X_2} = E(X_1 + X_2)^2 - [E(X_1 + X_2)]^2$

$(X_1 + X_2)^2 = (x_1 + x_2)^2$	$P(X_1 + X_2)^2 = (x_1 + x_2)^2$	**Col 1 x Col 2**
4	1/36	4/36
9	2/36	18/36
16	3/36	48/36
25	4/36	100/36
36	5/36	180/36
49	6/36	294/36
64	5/36	320/36
81	4/36	324/36
100	3/36	300/36
121	2/36	242/36
144	1/36	144/36
		$E[(X_1 + X_2)^2] = 1974/36$

Step 2: $[E(X_1 + X_2)]^2 = (\frac{7}{2} + \frac{7}{2})^2 = 49$

$$\sigma^2_{X_1+X_2} = E(X_1 + X_2)^2 - [E(X_1 + X_2)]^2 = 1974/36 - 49 = 35/6$$

$$\sigma^2_{X_1} + \sigma^2_{X_2} = \frac{35}{12} + \frac{35}{12} = \frac{35}{6}$$

➤e.

Step 1: $\sigma^2_{\bar{x}} = \frac{\sigma^2}{2} = \frac{\frac{35}{12}}{2} = \frac{35}{24}$

Therefore,

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{2}}$$

➤f.
Step 1:From the above table, $P\{X_1 > 3.5\} = (1/6) + (1/6) + (1/6) = 1/2$

Step 2:From the above table, $P\{\overline{X} > 3.5\} = 5/36 + 4/36 + 3/36 + 2/36 + 1/36 = 5/12$

6.
➤a.
We can define a binomial experiment of N independent trials as follows:

Let $S = X_1 + X_2 + ... + X_N$ where $\{X_k = 1\}$ is the event that on the kth trial success occurred and

$\{X_k = 0\}$ failure occurs.

$\mu = E(S) = E(X_1 + X_2 + ... + X_N) = E(X_1) + E(X_2) + ... + E(X_N) =$

$1p + 0(1 - p) + 1p + 0(1 - p) + ... + 1p + 0(1 - p) = Np.$

➤b.
$E\{X_pX_q\} = 1P\{X_p = 1\ X_q = 1\} + 0P\{X_p Xq = 0\} = 1P\{X_p = 1\}P\{X_q = 1\} = p^2$

$E(X_k^{\ 2}) = 1P\{X_k^{\ 2} = 1^2\} + 1P\{X_k^{\ 2} = 0^2\} = 1p$

$\sigma^2 = E(S^2) - E(S)^2 = E(S^2) - (Np)^2$

$E(S^2) = E[(X_1 + ... + X_N)^2] = E(X_1^2) + E(X_2^2) + ... + E(X_N^2) +$

$E(X_1X_2) + E(X_1X_3) + ... + E(X_1X_N) +$

$E(X_2X_1) + E(X_2X_3) + ... + E(X_2X_N) + E(X_NX_1) + E(X_NX_2) + ... + E(X_NX_{N-1}) =$

$Np + (N^2 - N)p^2$

$\sigma^2 = E(S^2) - E(S)^2 = E(S^2) - (Np)^2 = Np + (N^2 - N)p^2 - (Np)^2 = Np(1 - p)$

Therefore, $\sigma = \sqrt{Np(1-p)}.$

7.
$E(X) = x_1P\{X = x_1\} + x_2P\{X = x_2\} + ... + x_mP\{X = x_m\}$

$E(Y) = y_1P\{Y = y_1\} + y_2P\{Y = y_2\} + ... + y_nP\{Y = y_n\}$

$E(X)E(Y) =$

$[x_1P\{X = x_1\} + x_2P\{X = x_2\} + ... + x_mP\{X = x_m\}][y_1P\{Y = y_1\} + y_2P\{Y = y_2\} + ... + y_nP\{Y = y_n\}]$

=

$x_1y_1P\{X = x_1\}P\{Y = y_1\} + x_1y_2P\{X = x_1\}P\{Y = y_2\} + ... + x_1y_nP\{X = x_1\}P\{Y = y_n\} +$

$x_2y_1P\{X = x_2\}P\{Y = y_1\} + x_2y_2P\{X = x_2\}P\{Y = y_2\} + ... + x_2y_nP\{X = x_2\}P\{Y = y_n\} + ... +$

$x_my_1P\{X = x_m\}P\{Y = y_1\} + x_my_2P\{X = x_m\}P\{Y = y_2\} + ... + x_my_nP\{X = x_m\}P\{Y = y_n\} =$

$x_1y_1P\{X = x_1, Y = y_1\} + x_1y_2P\{X = x_1, Y = y_2\} + ... + x_1y_nP\{X = x_1, Y = y_n\} +$

$x_2y_1P\{X = x_2, Y = y_1\} + x_2y_2P\{X = x_2, Y = y_2\} + ... + x_2y_nP\{X = x_2, Y = y_n\} + ... +$

$x_my_1P\{X = x_m, Y = y_1\} + x_my_2P\{X = x_m, Y = y_2\} + ... + x_my_nP\{X = x_m, Y = y_n\} = E(XY)$

8.

➤a.

$E(X_k) = 1p_k + 0q_k = p_k$

Therefore, $E(S) = E(X_1 + X_2 + ... + X_N) = E(X_1) + E(X_2) + ... + E(X_N) = p_1 + p_2 + ... + p_N$

➤b.

$\sigma^2_{X_k} = E(X_k^2) - E(X_k)^2 = p_k - p_k^2 = p_kq_k$

Because the random variables are mutually independent,

$\sigma^2_S = \sigma^2_{X_1} + \sigma^2_{X_2} + ... + \sigma^2_{X_N} = p_1q_1 + p_2q_2 + ... + p_Nq_N.$

9.

➤a.

We assume $E(X_k) = \mu$

$\overline{X} = (X_1 + X_2 + ... + X_n)/n$

$E(\overline{X}) = E[(X_1 + X_2 + ... + X_n)/n)] = \{E(X_1) + ... + E(X_n)\}/n = n\mu/n = \mu$

➤b.

$$\sigma^2_{\overline{X}} = \frac{\sigma^2(X_1 + X_2 + ... + X_n)}{n^2} = \frac{\sigma^2_{X_1} + \sigma^2_{X_2} + ... + \sigma^2_{X_n}}{n^2} = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

Taking the square root of both sides we have $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}}$

10.

➤a.

$\mu = E(X) = 2P\{X = 2\} + 10P\{X = 10\} = 2(1/2) + 10(1/2) = 6$

$\sigma^2 = E(X^2) - E(X)^2 = 2^2P\{X^2 = 4\} + 10^2 P\{X^2 = 100\} - 6^2 = 2 + 50 - 36 = 16$

➤b.
Since the sample is of size N = 30 and taken with replacement, the size of the population $\overline{\boldsymbol{\Omega}}$.

2^{30} = 1,073,741,824

➤c.
Since there are only 2 numbers that can be selected, the general formula for each possible distinct value is [2(k) + 10(30- k)]/30 = (300 - 8k)/30 = 10 - 4k/15 where k = 0, 1,2, ...,30.

$\overline{X}$ = 300/30, 292/30, 284/30, 276/30, 268/30, 260/30, 252/30, 244/30,

236/30, 228/30, 220/30,212/30, 204/30,196/30,188/30,180/30,

172/30,164/30,156/30,148/30,140/30,132/30,124/30,116/30,108/30,

100/30, 92/30, 84/30, 76/30, 68/30, 60/20.

➤d.
The number of ways of selecting k 2s' and (30-k) 10s' where order is not important is

$\binom{30}{k}$.

Since, $P\{\overline{X} = 10 - 4k/15\} = \dfrac{\binom{30}{k}}{2^{30}} = \binom{30}{k}\dfrac{1}{2^k}\dfrac{1}{2^{30-k}}$, the distribution of $\overline{X}$ is binomial.

➤e.
From the summation formula for $E(\overline{X})$ =

$$\frac{\frac{300}{30}\binom{30}{0} + \frac{292}{30}\binom{30}{1} + \frac{284}{30}\binom{30}{2} + \ldots + \frac{300-8k}{30}\binom{30}{k} + \ldots + \frac{60}{30}\binom{30}{30}}{2^{30}} =$$

$$\frac{300\binom{30}{0} + 292\binom{30}{1} + 284\binom{30}{2} + \ldots + (300-8k)\binom{30}{k} + \ldots + 60\binom{30}{30}}{30(2^{30})}.$$

➤f.
From the central limit theorem,

$E(\overline{X}) = \mu = 6.$

➤g.
Step 1:
$\overline{X} = 10 - 4k/15$

$4.9 \le 10 - 4k/15 \le 6.8$

$-5.1 \le -4k/15 \le -3.2$

$3.2 \le 4k/15 \le 5.1$

$48 \le 4k \le 76.5$

$12 \le k \le 19.125$

Therefore, $12 \le k \le 19$.

Step 2: For these values of k, we will sum $P\{\overline{X} = 10 - 4k/15\}$, k = 12, 13, ...,19 :

$P\{4.9 \le \overline{X} \le 6.8\} \approx$

$P\{\overline{X} = 4.9\} + P\{\overline{X} \approx 5.17\} + P\{\overline{X} \approx 5.44\} + P\{\overline{X} \approx 5.71\} + P\{\overline{X} \approx 5.98\} + P\{\overline{X} \approx 6.25\} +$

$P\{\overline{X} = 6.52\} + P\{\overline{X} \approx 6.8\} =$

$$\frac{\binom{30}{12}}{2^{30}} + \frac{\binom{30}{13}}{2^{30}} + \frac{\binom{30}{14}}{2^{30}} + \frac{\binom{30}{15}}{2^{30}} + \frac{\binom{30}{16}}{2^{30}} + \frac{\binom{30}{17}}{2^{30}} + \frac{\binom{30}{18}}{2^{30}} + \frac{\binom{30}{19}}{2^{30}} \approx 0.85$$

➤h.
From the central limit theorem we have $\mu = 6$,

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}} = \frac{4}{\sqrt{30}} \approx 0.73,$$

$$z = \frac{6.8 - 6}{0.73} = 1.1 .$$

From the normal distribution table, we have the area 0.3643.

$$z = \frac{4.9 - 6}{0.73} \approx -1.51$$

From the normal distribution table, we have the area 0.4345 .

Therefore, according to the central limit theorem we have

$P\{4.9 \le \overline{X} \le 6.8\} \approx 0.3643 + 0.4345 \approx 0.80$.

11.

$$E(Z) = E[\frac{\overline{X} - \mu}{\sigma_{\overline{X}}}] = \frac{1}{\sigma_{\overline{X}}}E(\overline{X} - \mu) = \frac{1}{\sigma_{\overline{X}}}[E(\overline{X}) - E(\mu)] = \frac{1}{\sigma_{\overline{X}}}[\mu - \mu] = 0$$

$\sigma^2(Z) = E(Z^2) - [E(Z)]^2 = E(Z^2) - 0^2 = E(Z^2)$

$$E(Z^2) = E[\frac{\overline{X}^2 - 2\mu\overline{X} + \mu^2}{\sigma^2_{\overline{X}}}] = \frac{1}{\sigma^2_{\overline{X}}}E[\overline{X}^2 - 2\mu\overline{X} + \mu^2] = \frac{E[\overline{X}^2] - 2\mu E[\overline{X}] + \mu^2]}{\sigma^2_{\overline{X}}} =$$

$$\frac{E[\overline{X}^2] - E(\overline{X})^2}{\sigma^2_{\overline{X}}} = 1$$

12. $Z = \dfrac{\overline{X} - \mu}{\sigma_{\overline{X}}}$

where

$$\sigma_{\overline{X}} = \frac{s}{\sqrt{N}}.$$

$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$$

$$E(Z) = E[\frac{\overline{X} - \mu}{\sigma_{\overline{X}}}] = \frac{1}{\sigma_{\overline{X}}}E(\overline{X} - \mu) = \frac{1}{\sigma_{\overline{X}}}[E(\overline{X}) - E(\mu)] = \frac{1}{\sigma_{\overline{X}}}[\mu - \mu] = 0$$

Define $\sigma^* = \dfrac{\sigma}{\sqrt{N}}$.

$$Z = \frac{\sigma^*(\overline{X} - \mu)}{\sigma^*\sigma_{\overline{X}}} = \frac{\sigma^*}{\sigma_{\overline{X}}}\frac{(\overline{X} - \mu)}{\sigma^*}$$

$$\sigma^2(Z) = \frac{\sigma^{*2}}{\sigma_{\overline{X}}^2}\sigma^2[\frac{(\overline{X} - \mu)}{\sigma^*}] = \frac{\sigma^{*2}}{\sigma^2_{\overline{X}}} = \frac{\sigma^2}{s^2}$$

$$\sigma(Z) = \frac{\sigma}{s}$$

13.

➤a.

μ. = 1(0.51) + 2(0.23) + 3(0.17) + 4(0.09) = 1.84

➤b.

$\sigma^2 = 1^2(0.51) + 2^2(0.23) + 3^2(0.17) + 4^2(0.09) - (1.84)^2 \approx 4.4 - 3.39 = 1.01$

$\sigma \approx 1$

$\sigma_{\overline{X}} = 1/10 = 0.1$

➤c.

z = (2 - 1.84)/0.1 = 1.6

From Table C, $P\{0 \le Z \le 1.6\} = 0.4452$.

$P\{Z > 1.6\} = .5 - 0.4452 \approx 0.05$.

14.

➤a.

{1,2,3,4,5,6}

➤b.

μ = 1/6 + 2/6 + 3/6 + 4/6 + 5/6 + 6/6 = 3.5

Let X = k (k = 1, ..., 6) .

$\sigma^2 = E(X^2) - [E(X)]^2 = E(X^2) - 3.5^2$

$E(X^2) = 1/6 + 4/6 + 9/6 + 16/6 + 25/6 + 36/6 \approx 15.17$

$\sigma^2 = E(X^2) - [E(X)]^2 = 15.17 - 3.5^2 = 2.92$

$\sigma \approx 1.71$

➤c.

$$\sigma_{\overline{X}} = \frac{1.71}{\sqrt{64}} \approx 0.21$$

$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$$

z = (4 - 3.5)/0.21 = 0.5/0.21 = 2.38

$P(0 \le Z \le 2.38) = 0.4913$

$z = (5 - 3.5)/0.21 = 7.14$

$P(0 \le Z \le 7.14) = 0.4999$

$P(4 \le \overline{X} \le 5) \approx 0.01$

15.

➤a.

{01,2,3,4,5}

➤b.

X = k ; k = 0,1,2,3,4,5.

$$P(X = k) = \frac{\binom{13}{k}\binom{39}{5-k}}{\binom{52}{5}}$$

$$\mu = E(X) = (0)\frac{\binom{13}{0}\binom{39}{5-0}}{\binom{52}{5}} + (1)\frac{\binom{13}{1}\binom{39}{5-1}}{\binom{52}{5}} + (2)\frac{\binom{13}{2}\binom{39}{5-2}}{\binom{52}{5}} + (3)\frac{\binom{13}{3}\binom{39}{5-3}}{\binom{52}{5}} +$$

$$(4)\frac{\binom{13}{4}\binom{39}{5-4}}{\binom{52}{5}} + (5)\frac{\binom{13}{5}\binom{39}{5-5}}{\binom{52}{5}} =$$

[1,069,263 + 1,425,684 + 635,778 + 111,540 + 6435]/2,598,960 = 3,248,700/2,598,960 = 1.25

$\sigma^2 = E(X^2) - [E(X)]^2 = E(X^2) - 1.25^2$

$$E(X^2) = (0^2)\frac{\binom{13}{0}\binom{39}{5-0}}{\binom{52}{5}} + (1^2)\frac{\binom{13}{1}\binom{39}{5-1}}{\binom{52}{5}} + (2^2)\frac{\binom{13}{2}\binom{39}{5-2}}{\binom{52}{5}} + (3^2)\frac{\binom{13}{3}\binom{39}{5-3}}{\binom{52}{5}} +$$

$$(4^2)\frac{\binom{13}{4}\binom{39}{5-4}}{\binom{52}{5}}+(5^2)\frac{\binom{13}{5}\binom{39}{5-5}}{\binom{52}{5}} =$$

[1,069,263 + 2,851,368 + 1,907,334 + 446,160 + 32175]/2,598,960 $\approx$ 2.43

$\sigma^2 = E(X^2) - [E(X)]^2 = E(X^2) - 1.25^2 = 2.43 - 1.25^2 = 0.87$

$\sigma \approx 0.93$

➤c.

$$\sigma_{\overline{X}} = \frac{0.93}{\sqrt{100}} \approx 0.09$$

$z = (2 - 1.25)/0.09 = 8.33$

$P(\overline{X} < 2) = P(Z < 8.33) \approx 1$