Probability theory
Lesson 23
The Poisson Distribution Table

23.1- Using The Poisson Time Distribution Table.

23.1 - Problem 1:

➤(a).

To use the table we must first find μ.

Step 1: $\mu t = 3$, average number of students at a local university that violate curfew restrictions.

Step 2: Since the time period is 2 hours, $t = 2$.

Step 3: Since the unit of time is 2 hours, we redefine $\mu = 3$

The probability that at least 3 students violate curfew equals

$P\{X \geq 3\} = 1 - P\{X \leq 2\}$, the values given in the table.

Step 4: From the table, check the column $\mu = 3$.

Step 5: From the table, check the row $x = 2$.

Step 6: From the table, the intersection of the row and column gives

$P\{X \leq 2\} \approx 0.4232$.

Step 7: $P\{X \geq 3\} = 1 - P\{X \leq 2\} \approx 1 - 0.4232 = 0.5768$.

➤(b).

Step 1: $P\{X \leq 6\}$, the probability that at most 6 students violates curfew.

Step 2: From the table, check the column $\mu = 3$.

Step 3: From the table, check the row $x = 6$.

Step 4: From the table, the intersection of the row and column gives

$P\{X \leq 6\} \approx 0.9665$.

➤(c).
Step 1: $P\{2 \le X \le 7\}$, the probability between 2 and 7 students violates curfew.

Step 2: $P\{2 \le X \le 7\} = P\{X \le 7\} - P\{X \le 1\}$

Step 3: From the table, check the column $\mu = 3$ and the row $x = 7$.

Step 4: The intersection of the row and column gives

$P\{X \le 7\} \approx 0.9989$

Step 5: From the table, check the column $\mu = 3$ and the row $x = 1$.

Step 4: The intersection of the row and column gives

$P\{X \le 1\} \approx 0.1991$.

Step 5: Therefore,

$P\{2 \le X \le 7\} = P\{X \le 7\} - P\{X \le 1\} \approx 0.9989 - 0.1991 = 0.7998$

23.1 - Problem 2:
$k = 5$, since we are interested in the probability that 5 customers request an oil change. The time period we are concerned is $t = 3$ hours. Therefore we need to compute μ.

Step 1: Over 2 hours, the average is 3 requests. Therefore,

$2\mu = 3$ and

$\mu = 3/2 = 1.5$

the average number of requests per hour.

Step 2: Since $t = 3$, we have

$t\mu = 3(1.5) = 4.5$, the average number of requests over 3 hours.

Step 3: To use the table, we redefine $\mu = 4.5$.

Step 4: $P\{X = 5\} = P\{X \le 5\} - P\{X \le 4\}$, probability that 5 requests occur over three hours.

Step 5: Going to the column for $\mu = 4.5$ from the table, we have

$P\{X \le 5\} \approx 0.7029$

$P\{X \le 4\} \approx 0.5321$

Step 6: $P\{X = 5\} = P\{X \le 5\} - P\{X \le 4\} \approx 0.7029 - 0.5321 = 0. = 0.1708$.

23.1 - Problem 3:

➤(a).

Step 1: Assume 10% of all the sets in the shipment are defective.

$\mu = 25(0.10) = 2.5$

Step 2: From the Poisson table,

$P\{X < 4\} = P\{X \le 3\} = 0.7576$

➤(b).

Step 1: Assume 2% of all sets in the shipment are defective.

$\mu = 25(0.02) = 0.5$.

Step 2: From the Poisson table,

$P\{X \ge 4\} = 1 - P\{X \le 3\} = 1 - 0.9982 = 0.0018$.

Supplementary Problems

1.

Step 1: The following table is copied from the Poisson Distribution table for $P\{X \le x\}$

μ =	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**
x					
0	.3679	.1353	.0498	.0183	.0067
1	.7358	.4060	.1991	.0916	.0404
2	.9197	.6767	.4232	.2381	.1247
3	.9810	.8571	.6472	.4335	.2650
4	.9963	.9473	.8153	.6288	.4405
5	.9994	.9834	.9161	.7851	.6160

From this table we use P{X ≥ x} = 1 - P{X ≤ x - 1}

μ =	**1**	**2**	**3**	**4**	**5**
x					
0	1	1	1	1	1
1	1 - .3679 = 0.6321	1 - .1353 = 0.8647	1 -.0498 = 0.9502	1 -.0183 = 0.9817	1 -.0067 = 0.9933
2	1 -.7358 = 0.2642	1 - .4060 = 0.5940	1 -.1991 = 0.8009	1 -.0916 = 0.9084	1 -.0404 = 0.9596
3	1 -.9197 = 0.0803	1 - .6767 = 0.3233	1 -.4232 = 0.5768	1 - .2381= 0 .7619	1 -.1247 = 0.8753
4	1 -.9810 = 0.0190	1 - .8571 = 0.1429	1 -.6472 = 0.3528	1 -.4335 = 0.5665	1 -.2650 = 0.7350
5	1 -.9963 = .00370	1 -.9473 = 0 .0527	1 -.8153 = 0.1847	1 -.6288 = 0 .3712	1 -.4405 = 0.5595

This gives P{X ≥ x}

μ =	**1**	**2**	**3**	**4**	**5**
x					
0	1.000	1.000	1.000	1.000	1.000
1	0.6321	0.8647	0.9502	0.9817	0.9933
2	0.2642	0.5940	0.8009	0.9084	0.9596
3	0.0803	0.3233	0.5768	0.7619	0.8753
4	0.0190	0.1429	0.3528	0.5665	0.7350
5	0.0037	0.0527	0.1847	0.3712	0.5595

2.
Since the basic unit is 1,000 words, then μ = 5. Therefore, we find from the table

P{X ≥ 4} = 1 - P{X ≤ 3} ≈ 1 - 0.2650 = 0.7350.

3.
Since the basic unit is 100 square feet of coverage, then for 200 square feet of coverage μ = 4.

P{2 ≤ X ≤ 5} = P{X ≤ 5} - P{X ≤ 1} ≈ 0.7851 - 0.0916 = 0.6935

4.
➤a.
Since the basic unit is 1,000, we have 0.015(1,000) = 15. Since the sample is 100, we have

$\mu = 1.5$.

From the table,

$P\{X \geq 5\} \approx 1 - P\{X \leq 4\} = 1 - 0.9814 = 0.0186$.

➤b.
Since the basic unit is 1,000, we have 0.05(1,000) = 50. Since the sample is 100, we have $\mu = 5$.

From the table,

$P\{X \leq 4\} \approx 0.4405$

5.
Step 1: Since the basic unit is the sample of 50 chips, then

$\mu = 0.04(50) = 2$.

Step 2: The probability that only 50 chips are inspected is taken from the table for

$P\{X \leq 4\} \approx 0.9473$, the probability that only 50 chips are inspected.

$P\{X \geq 5\} \approx 1 - 0.9473 = 0.0527$.

Step 3: If X represents the random variable of the number of chips per inspected, we have

$E(X) = 50(0.9473) + 100(0.0527) = 52.63$, average number of chips inspected.