PROBABILITY THEORY
Lesson 13
Bayes's Theorem

13.1 - What is Bayes's Theorem ?

13.1 - Problem 1:
B: The event that the student will earn at least a B in the course.

F: The event that the student will study for the final.

From the problem we are given:

P(**B**|**F**) = 0.35: If a student studies for the final, he/she has a 35% chance of earning at least a B in the course.

P(**F**) = 0.50: there is a 50% chance that the student will study for the final

P(**B**) = 0.28: there is a 28% chance that the student will earn at least a B in the course.

From the end of problem, we need to find:

Find P(**F**|**B**), the probability that if the student earned at least a B in the course then he/she studied for the final.

$$P(\mathbf{F}|\mathbf{B}) = \frac{P(\mathbf{B}|\mathbf{F})P(\mathbf{F})}{P(\mathbf{B})} = \frac{(0.35)(0.50)}{0.28} = 0.625$$

13.1 - Problem 2:
$\mathbf{R_1}'$: The event that a red marble was not selected from urn 1.

$\mathbf{R_2}$**:** The event that a red marble was selected from urn 2.

From the problem we are given:

Find P($\mathbf{R_1}$) = 7/10, the probability of drawing a red marble from urn 1.

P($\mathbf{R_2}$|$\mathbf{R_1}'$) = 0: The probability that if a red marble is not selected from urn 1 and placed in urn 2, then a red marble is selected from urn 2.

From the end of problem, we need to find:

P($\mathbf{R_1}'$|$\mathbf{R_2}$): The probability that if a red marble is selected from urn 2, then a red marble was not selected from urn 1.

$$P(\mathbf{R_1}'|\mathbf{R_2}) = \frac{P(\mathbf{R_2}|\mathbf{R_1}')P(\mathbf{R_1}')}{P(\mathbf{R_2})} = \frac{(\frac{0}{6})(\frac{7}{10})}{P(\mathbf{R_2})} = 0.$$

13.1 - Problem 3:

$\mathbf{S_1}$: The event that the winning student came from section 1.

$\mathbf{S_2}$: The event that the winning student came from section 2.

$\mathbf{S_3}$: The event that the winning student came from section 3.

F: The event that the winning student is a female.

Find $P(\mathbf{S_1}|\mathbf{F})$, assuming the winner is a female, the probability that the winner was selected from section 1.

$$\mathbf{P(S_1|F)} = \frac{P(\mathbf{F}|\mathbf{S_1})P(\mathbf{S_1})}{P(\mathbf{F})} = \frac{(\frac{25}{75})(\frac{75}{225})}{P(\mathbf{F})}$$

$$\mathbf{F} = (\mathbf{F}\cap\mathbf{S_1})\cup(\mathbf{F}\cap\mathbf{S_2})\cup(\mathbf{F}\cap\mathbf{S_3})$$

$$P(\mathbf{F}) = P[(\mathbf{F}\cap\mathbf{S_1})\cup(\mathbf{F}\cap\mathbf{S_2})\cup(\mathbf{F}\cap\mathbf{S_3})] = P(\mathbf{F}\cap\mathbf{S_1}) + P(\mathbf{F}\cap\mathbf{S_2}) + P(\mathbf{F}\cap\mathbf{S_3}) =$$

$$P(\mathbf{S_1})P(\mathbf{F}|\mathbf{S_1}) + P(\mathbf{S_2})P(\mathbf{F}|\mathbf{S_2}) + P(\mathbf{S_3})P(\mathbf{F}|\mathbf{S_3}) = (\frac{75}{225})(\frac{25}{75}) + (\frac{100}{225})(\frac{60}{100}) + (\frac{50}{225})(\frac{20}{50}) = \frac{7}{15}$$

$$\mathbf{P(S_1|F)} = \frac{P(\mathbf{F}|\mathbf{S_1})P(\mathbf{S_1})}{P(\mathbf{F})} = \frac{(\frac{25}{75})(\frac{75}{225})}{P(\mathbf{F})} = \frac{(\frac{25}{75})(\frac{75}{225})}{(\frac{7}{15})} = \frac{5}{21}$$

13.2 - Real Life Applications

13.2 - Problem 1:

➤(a).

I: the event that a driver is intoxicated.

A: the event that a driver will get involved in a fatal auto accident.

$P(\mathbf{A}) = 0.09$

$P(\mathbf{I}|\mathbf{A}) = 0.35$

$P(\mathbf{I}) = 0.05$

We want to find $P(\mathbf{A}|\mathbf{I})$.

$$P(\mathbf{A}|\mathbf{I}) = \frac{P(\mathbf{I}|\mathbf{A})P(\mathbf{A})}{P(\mathbf{I})} = \frac{(0.35)(0.09)}{0.05} = 0.63$$

➤(b).
We want to find $P(\mathbf{A}'|\mathbf{I}) = 1 - P(\mathbf{A}|\mathbf{I}) = 1 - 0.63 = 0.37$.

➤(c).

$$P(\mathbf{A}|\mathbf{I}') = \frac{(1-P(\mathbf{I}|\mathbf{A}))P(\mathbf{A})}{1 - P(\mathbf{I})} = \frac{(0.65)(0.09)}{0.95} \approx 0.06$$

13.2 Problem 2:

➤(a).
Step 1: **D**: The event that his patients are diabetics.

A: The event that a patient tests positive

Step 2: $P(\mathbf{D}) = 0.10$, the probability that a patient has diabetes.

$P(\mathbf{A}|\mathbf{D}) = 0.90$, the probability, that given a patient has diabetes, he/she correctly tests positive.

$P(\mathbf{A}|\mathbf{D}') = 0.15$, the probability, that given a patient does not have diabetes, he/she incorrectly tests positive.

Find $P(\mathbf{D}|\mathbf{A})$, assuming a patient tests positive, the probability that he/she has diabetes.

Step 3: $\mathbf{P(D|A)} = \dfrac{P(\mathbf{A}|\mathbf{D})P(\mathbf{D})}{P(\mathbf{A})} = \dfrac{(0.90)(0.10)}{P(\mathbf{A})}$

Step 4: $\mathbf{A} = (\mathbf{A}\cap\mathbf{D})\cup(\mathbf{A}\cap\mathbf{D}')$, the event the patient tests positive and has diabetes or the patient tests positive and does not have diabetes.

$P(\mathbf{A}) = P[(\mathbf{A}\cap\mathbf{D})\cup(\mathbf{A}\cap\mathbf{D}')] = P(\mathbf{A}\cap\mathbf{D}) + P(\mathbf{A}\cap\mathbf{D}') = P(\mathbf{D})P(\mathbf{A}|\mathbf{D}) + P(\mathbf{D}')P(\mathbf{A}|\mathbf{D}') =$

$(0.10)(0.90) + (0.90)(0.15) = 0.225$

Step 5: $\mathbf{P(D|A)} = \dfrac{(P(\mathbf{A}|\mathbf{D})P(\mathbf{D})}{P(\mathbf{A})} = \dfrac{(0.90)(0.10)}{(0.225)} \approx 0.4$

➤(b).
$\mathbf{D}'$ is the event that the patient does not have diabetes. Therefore, if a patient tests positive. the probability that he or she does not have diabetes is $\mathbf{P(D|A)} = 1 - 0.40 = 0.60$.

➤(c).

$$\mathbf{P(D|A')} = \frac{(1 - P(\mathbf{A}|\mathbf{D}))P(\mathbf{D})}{1 - P(\mathbf{A})} = \frac{(0.10)(0.10)}{(0.775)} \approx 0.01$$

13.2 - Problem 3:

➤(a).

Step 1:

N: The event that sales of new shoes exceeds \$500,000.

A: The event that the new store will be successful.

Step 2:

P(**N**) = 0.80, the probability that sales of new shoes exceeds \$500,000.

P(**A**|**N**) = 0.70, the probability, that if sales of new shoes exceeds \$500,000, the new store will be successful.

P(**A**′|**N**) = 0.30, the probability, that if sales of new shoes exceeds \$500,000, the new store will not be successful.

P(**A**|**N**′) = 0.25, the probability, that if sales of new shoes does not exceeds \$500,000, the new store will be successful.

P(**A**′|**N**′) = 0.75, the probability, that if sales of new shoes does not exceeds \$500,000, the new store will not be successful.

Find P(**N**|**A**′), assuming the new store is not successful, the probability that the sales in the area exceed \$500,000.

Step 3: $\mathbf{P(N|A')} = \dfrac{P(\mathbf{A}'|\mathbf{N})P(\mathbf{N})}{P(\mathbf{A}')} = \dfrac{(0.30)(0.80)}{P(\mathbf{A}')}$

Step 4: **A**′ = (**A**′∩**N**)∪(**A**′∩**N**′), the event the store is not successful and sales of new shoes exceed \$500,000 and the store is not successful and sales of new shoes do not exceed \$500,000.

$P(\mathbf{A}') = P[(\mathbf{A}'\cap\mathbf{N})\cup(\mathbf{A}'\cap\mathbf{N}')] = P(\mathbf{A}'\cap\mathbf{N}) + P(\mathbf{A}'\cap\mathbf{N}') = P(\mathbf{N})P(\mathbf{A}'|\mathbf{N}) + P(\mathbf{N}')P(\mathbf{A}'|\mathbf{N}') =$

$(0.80)(0.30) + (0.20)(0.75) = 0.39.$

Step 5: $\mathbf{P(N|A')} = \dfrac{P(\mathbf{A}'|\mathbf{N})P(\mathbf{N})}{P(\mathbf{A}')} = \dfrac{(0.30)(0.80)}{(0.39)} \approx 0.62.$

➤(b).

D′ is the event that sales in the area will not exceed \$500,000. Therefore, P(**D**|**A**) = 1 - 0.62 = 0.38.

➤(c)

$$\mathbf{P(N|A)} = \frac{(1 - P(\mathbf{A}|\mathbf{N}))P(\mathbf{N})}{1 - P(\mathbf{A})} = \frac{(0.70)(0.80)}{(0.61)} \approx 0.83$$

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13.2 - Problem 4:

➤(a).

Step 1:

A: the event that the 50% of the stores will return trains.

B: the event that the 70% of the stores will return trains.

C: the event that the 80% of the stores will return trains.

D: the event that the 90% of the stores will return trains.

R: the event that the store selected has returned some of the trains.

Step 2:
P(**A**) = 0.60, the probability that 50% of the stores will return some trains.

P(**B**) = 0.15, the probability that 70% of the stores will return some trains.

P(**C**) = 0.12, the probability that 80% of the stores will return some trains.

P(**B**) = 0.13, the probability that 90% of the stores will return some trains.

P(**R**|**A**) = 0.50, given the event that 50% of the stores will return trains, the probability is 0.50 that the store selected has returned some trains.

P(**R**|**B**) = 0.70, given the event that 70% of the stores will return trains, the probability is 0.70 that the store selected has returned some trains.

P(**R**|**C**) = 0.80, given the event that 80% of the stores will return trains, the probability is 0.80 that the store selected has returned some trains.

P(**R**|**D**) = 0.90, given the event that 90% of the stores will return trains, the probability is 0.90 that the store selected has returned some trains.

Find P(**B**|**R**), given that a store selected returned some trains, the probability that 70% of the stores will return trains.

Step 3: $P(\mathbf{B}|\mathbf{R}) = \dfrac{P(\mathbf{R}|\mathbf{B})P(\mathbf{B})}{P(\mathbf{R})} = \dfrac{(0.70)(0.15)}{P(\mathbf{R})}$

Step 4: **R** = (**R**∩**A**)∪(**R**∩**B**)∪(**R**∩**C**)∪(**R**∩**D**), the event that the store selected has returned some of the trains and 50% of the stores will return trains

or
the store selected has returned some of the trains and 70% of the stores will return trains

or
the store selected has returned some of the trains and 80% of the stores will return trains

or
the store selected has returned some of the trains and 90% of the stores will return trains.

$P(\mathbf{R}) = P[(\mathbf{R}\cap\mathbf{A})\cup(\mathbf{R}\cap\mathbf{B})\cup(\mathbf{R}\cap\mathbf{C})\cup(\mathbf{R}\cap\mathbf{D})] = P(\mathbf{R}\cap\mathbf{A}) + P(\mathbf{R}\cap\mathbf{B}) + P(\mathbf{R}\cap\mathbf{C}) + P(\mathbf{R}\cap\mathbf{D}) =$

$P(\mathbf{A})P(\mathbf{R}|\mathbf{A}) + P(\mathbf{B})P(\mathbf{R}|\mathbf{B}) + P(\mathbf{C})P(\mathbf{R}|\mathbf{C}) + P(\mathbf{D})P(\mathbf{R}|\mathbf{D}) =$

(0.60)(0.50) + (0.15)(0.70) + (0.13)(0.80) + (0.12)(0.90) = 0.617

Step 5: $\mathbf{P(B|R)} = \dfrac{P(\mathbf{R}|\mathbf{B})P(\mathbf{B})}{P(\mathbf{R})} = \dfrac{(0.70)(0.15)}{0.617} \approx 0.17$

➤(b).
B′ is the event that 70% of the stores will not return the unsold trains.

Therefore, **P**(**B**|**R**) = 1 - 0.17 = 0.83.

Supplementary Problems

1.
Step 1: Solution from the table.

P(**M**|**J**) = P(**M**∩**J**)/P(**J**)

#**S** = 185, the total number of students.

#(**M**∩**J**) = 50, the number of males that like jazz.

#**J** = 110, the number of students that like jazz.

P(**M**∩**J**) = 50/185

P(**J**) = 110/185

P(**M**|**J**) = P(**M**∩**J**)/P(**J**) = 5/11.

Step 2: Solution using Bayes Theorem.

$\mathbf{P(M|J)} = \dfrac{\mathbf{P(J|M)P(M)}}{\mathbf{P(J)}}$

From the table, we see

P(**J**|**M**) = 50/85

P(**M**) = 80/185

P(**J**) = 110/185

$\mathbf{P(M|J)} = \dfrac{\mathbf{P(J|M)P(M)}}{\mathbf{P(J)}} = \dfrac{(\frac{50}{85})(\frac{85}{185})}{\frac{110}{185}} = \dfrac{5}{11}$

2.
Step 1: **I**: The event that the person is innocent of the crime.

G: The event that the jury finds the person guilty

Step 2: P(**I**) = 0.25, the probability that the person is really innocent.

$P(\mathbf{G}'|\mathbf{I}) = 0.90$, the probability if the person is innocent, the jury will find the accused not guilty.

$P(\mathbf{G}|\mathbf{I}') = 0.70$, the probability if the person is guilty of the crime, the jury will find the accused guilty.

$P(\mathbf{G}'|\mathbf{I}') = 0.30$, the probability if the person is guilty of the crime, the jury will find the accused not guilty.

Find $P(\mathbf{I}|\mathbf{G}')$, if a defendant is found not guilty, the probability that he/she is really innocent.

Step 3: $\mathbf{P(I|G')} = \dfrac{P(\mathbf{G}'|\mathbf{I})P(\mathbf{I})}{P(\mathbf{G}')} = \dfrac{(0.90)(0.25)}{P(\mathbf{G}')}$

Step 4: $\mathbf{G}' = (\mathbf{G}'\cap\mathbf{I})\cup(\mathbf{G}'\cap\mathbf{I}')$

$P(\mathbf{G}') = P[(\mathbf{G}'\cap\mathbf{I})\cup(\mathbf{G}'\cap\mathbf{I}')] = P(\mathbf{G}'\cap\mathbf{I}) + P(\mathbf{G}'\cap\mathbf{I}') = P(\mathbf{I})P(\mathbf{G}'|\mathbf{I}) + P(\mathbf{I}')P(\mathbf{G}'|\mathbf{I}') =$

$(0.25)(0.90) + (0.75)(0.30) = 0.45$

Step 5: $\mathbf{P(I|G')} = \dfrac{P(\mathbf{G}'|\mathbf{I})P(\mathbf{I})}{P(\mathbf{G}')} = \dfrac{(0.90)(0.25)}{0.45} = 0.5$

3.
Step 1:
J: the event he asks Jane out.
J′: the event he asks Sally out.
D: the event Sam gets a date.
Step 2: P(**J**) = 0.40

$P(\mathbf{D}|\mathbf{J}) = 0.75$, the probability that if Jane is asked out she accepts.

$P(\mathbf{D}|\mathbf{J}') = 0.30$, the probability that if Sally is asked out she accepts.

Find $P(\mathbf{J}'|\mathbf{D})$, assuming that Sam had a date last night, the probability that he was with Sally.

Step 3: $P(\mathbf{J}'|\mathbf{D}) = \dfrac{P(\mathbf{D}|\mathbf{J}')P(\mathbf{J}')}{P(\mathbf{D})} = \dfrac{(0.3)(0.6)}{P(\mathbf{D})}$

$\mathbf{D} = (\mathbf{D}\cap\mathbf{J}')\cup(\mathbf{D}\cap\mathbf{J})$

$P(\mathbf{D}) = P[(\mathbf{D}\cap\mathbf{J}')\cup(\mathbf{D}\cap\mathbf{J})] = P(\mathbf{D}\cap\mathbf{J}') + P(\mathbf{D}\cap\mathbf{J}) = P(\mathbf{J}')P(\mathbf{D}|\mathbf{J}') + P(\mathbf{J})P(\mathbf{D}|\mathbf{J}) =$

(0.60)(0.30) + (0.40)(0.75) = 0.49

$$P(\mathbf{J}'|\mathbf{D}) = \frac{P(\mathbf{D}|\mathbf{J}')P(\mathbf{J}')}{P(\mathbf{D})} = \frac{(0.3)(0.6)}{0.49} \approx 0.37$$

4.
Step 1:
D: the event that an American dies from lung cancer.

A: the event that an American is a smoker.

$P(\mathbf{D}) = \dfrac{400{,}000}{250{,}000{,}000} = \dfrac{1}{625} = 0.0016$, the probability that an American dies of lung cancer.

$P(\mathbf{A}) = \dfrac{50{,}000{,}000}{250{,}000{,}000} = \dfrac{1}{5} = 0.20$, the probability that an American smokes.

$P(\mathbf{A}|\mathbf{D}) = \dfrac{320{,}000}{400{,}000} = \dfrac{4}{5} = 0.80$, the probability that given an American dies from lung cancer, the person smokes.

Find $P(\mathbf{D}|\mathbf{A})$, the probability that if an American is a smoker, then the person will die of lung cancer.

Step 3: We need to find $P(\mathbf{A}|\mathbf{D}')$.

$$P(\mathbf{A}|\mathbf{D}') = \frac{\#(\mathbf{A}\cap\mathbf{D}')}{\#\mathbf{D}'}$$

$\#(\mathbf{A}\cap\mathbf{D}') = 50{,}000{,}000 - 320{,}000 = 49{,}680{,}000$, the number of Americans that smoke but have not died from lung disease.

$\#\mathbf{D}' = 250{,}000{,}000 - 400{,}000 = 249{,}600{,}000$

$$P(\mathbf{D}|\mathbf{A}) = \frac{P(\mathbf{A}|\mathbf{D})P(\mathbf{D})}{P(\mathbf{A})} = \frac{(0.80)(0.0016)}{P(\mathbf{A})}$$

$$P(\mathbf{A}|\mathbf{D}') = \frac{\#(\mathbf{A}\cap\mathbf{D}')}{\#\mathbf{D}'} = \frac{49{,}680{,}000}{249{,}600{,}000} = \frac{207}{1040} \approx 0.20$$

$\mathbf{A} = (\mathbf{A}\cap\mathbf{D})\cup(\mathbf{A}\cap\mathbf{D}')$

$P(\mathbf{A}) = P[(\mathbf{A}\cap\mathbf{D})\cup(\mathbf{A}\cap\mathbf{D}')] = P(\mathbf{A}\cap\mathbf{D}) + P(\mathbf{A}\cap\mathbf{D}') = P(\mathbf{D})P(\mathbf{A}|\mathbf{D}) + P(\mathbf{D}')P(\mathbf{A}|\mathbf{D}') =$

$(0.0016)(0.80) + (0.9984)(0.20) = 0.00128 + 0.19969 \approx 0.20$

$P(\mathbf{D}|\mathbf{A}) = \dfrac{P(\mathbf{A}|\mathbf{D})P(\mathbf{D})}{P(\mathbf{A})} = \dfrac{(0.80)(0.0016)}{0.20}$ = 0.0064, that probability that if an American smokes, he\she died of lung cancer.

5.

Step 1: $P(\mathbf{A}|\mathbf{B}) = \dfrac{P(\mathbf{A}\cap\mathbf{B})}{P(\mathbf{B})}$

$P(\mathbf{B}|\mathbf{A}) = \dfrac{P(\mathbf{A}\cap\mathbf{B})}{P(\mathbf{A})}$

Step 2: $P(\mathbf{A}|\mathbf{B}) = \dfrac{P(\mathbf{A}\cap\mathbf{B})}{P(\mathbf{B})} = P(\mathbf{B}|\mathbf{A}) = \dfrac{P(\mathbf{A}\cap\mathbf{B})}{P(\mathbf{A})}$

Therefore, $P(\mathbf{A}) = P(\mathbf{B})$.

6.

➤ a.

Let **D** be the event that the offspring has the disease.

Let **F** be the event that the father has the disease.

Let **M** be the event that the mother has the disease.

$P(\mathbf{F}) = 0.05$

$P(\mathbf{M}) = 0.07$

$P[\mathbf{D}|(\mathbf{F}\cap\mathbf{M}')] = 0.35$

$P[\mathbf{D}|(\mathbf{F}'\cap\mathbf{M}] = 0.55$

$P[\mathbf{D}|(\mathbf{F}\cap\mathbf{M}] = 0.80$

$P[\mathbf{D}|(\mathbf{F}'\cap\mathbf{M}'] = 0.02$

Because it is reasonable to assume the events **M** and **F** are independent,

$P(\mathbf{M}\cap\mathbf{F}) = P(\mathbf{M})P(\mathbf{F}) = (0.07)(0.05) = 0.0035.$

$P(\mathbf{M}'\cap\mathbf{F}) = P(\mathbf{M}')P(\mathbf{F}) = (0.93)(0.05) = 0.0465$

$P(\mathbf{M}\cap\mathbf{F}') = P(\mathbf{M})P(\mathbf{F}') = (0.07)(0.95) = 0.0665$

$P(\mathbf{M}'\cap\mathbf{F}') = P(\mathbf{M}')P(\mathbf{F}') = (0.93)(0.95) = 0.8835$

$\mathbf{D} = [\mathbf{D}\cap(\mathbf{F}\cap\mathbf{M}')]\cup[\mathbf{D}\cap(\mathbf{F}'\cap\mathbf{M})]\cup[\mathbf{D}\cap(\mathbf{F}\cap\mathbf{M})]\cup[\mathbf{D}\cap(\mathbf{F}'\cap\mathbf{M}')]$

$P(\mathbf{D}) = P[\mathbf{D}\cap(\mathbf{F}\cap\mathbf{M}')] + P[\mathbf{D}\cap(\mathbf{F}'\cap\mathbf{M})] + P[\mathbf{D}\cap(\mathbf{F}\cap\mathbf{M})] + P[\mathbf{D}\cap(\mathbf{F}'\cap\mathbf{M}')] =$

$P[\mathbf{D}|(\mathbf{F}\cap\mathbf{M}')]P(\mathbf{F}\cap\mathbf{M}') + P[\mathbf{D}|(\mathbf{F}'\cap\mathbf{M})]P(\mathbf{F}'\cap\mathbf{M}) + P[\mathbf{D}|(\mathbf{F}\cap\mathbf{M})]P(\mathbf{F}\cap\mathbf{M}) +$

$P[\mathbf{D}|(\mathbf{F}'\cap\mathbf{M}')]P(\mathbf{F}'\cap\mathbf{M}') =$

$(0.35)(0.0465) + (0.55)(0.0665) + (0.80)(0.0035) + (0.02)(0.8835) \approx 0.07$

We can write the problem as $P(\mathbf{F}\cap\mathbf{M}'|\mathbf{D})$.

Using Bayes formula:

$$P(\mathbf{F}\cap\mathbf{M}'|\mathbf{D}) = \frac{P(\mathbf{D}|\mathbf{F}\cap\mathbf{M}')P(\mathbf{F}\cap\mathbf{M}')}{P(\mathbf{D})} = \frac{(0.35)(0.0465)}{0.07} \approx 0.23$$

➤ b.

$$P(\mathbf{F}'\cap\mathbf{M}|\mathbf{D}) = \frac{P(\mathbf{D}|\mathbf{F}'\cap\mathbf{M})P(\mathbf{F}'\cap\mathbf{M})}{P(\mathbf{D})} = \frac{(0.55)(0.0665)}{0.07} \approx 0.52$$

➤ c.

$$P(\mathbf{F}\cap\mathbf{M}|\mathbf{D}) = \frac{P(\mathbf{D}|\mathbf{F}\cap\mathbf{M})P(\mathbf{F}\cap\mathbf{M})}{P(\mathbf{D})} = \frac{(0.80)(0.0035)}{0.07} = 0.04$$

➤ d.

$$P(\mathbf{F}'\cap\mathbf{M}'|\mathbf{D}) = \frac{P(\mathbf{D}|\mathbf{F}'\cap\mathbf{M}')P(\mathbf{F}'\cap\mathbf{M}')}{P(\mathbf{D})} = \frac{(0.02)(0.8835)}{0.07} \approx 0.25$$

➤ e.

$\mathbf{F} = (\mathbf{F}\cap\mathbf{M})\cup(\mathbf{F}\cap\mathbf{M}')$

$P(\mathbf{F}|\mathbf{D}) = P[(\mathbf{F}\cap\mathbf{M})\cup(\mathbf{F}\cap\mathbf{M}')|\mathbf{D}] = P[\mathbf{F}\cap\mathbf{M}|\mathbf{D}] + P[\mathbf{F}\cap\mathbf{M}'|\mathbf{D}] = 0.04 + 0.23 = 0.27$

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$\mathbf{R}_a$: A marble is selected from urn A.
$\mathbf{R}_b$: A marble is selected from urn B.
$\mathbf{R}_c$: A marble is selected from urn C.

A: Urn A is selected.
B: Urn B is selected.
C: Urn C is selected.

$\mathbf{U} = (\mathbf{A}_1\cap\mathbf{B}_2)\cup(\mathbf{B}_1\cap\mathbf{A}_2)$, the event that urns A and B were selected.

$\mathbf{R} = \mathbf{R}_1\cap\mathbf{R}_2$, the event that both marbles are red.

We wish to find $P(\mathbf{U}|\mathbf{R})$.

Using Bayes, formula, $P(\mathbf{U}|\mathbf{R}) = P(\mathbf{R}|\mathbf{U})P(\mathbf{U})/P(\mathbf{R})$.

Step 1: $\mathbf{U} = (\mathbf{A}_1 \cap \mathbf{B}_2) \cup (\mathbf{B}_1 \cap \mathbf{A}_2)$

$P(\mathbf{U}) = P(\mathbf{A}_1 \cap \mathbf{B}_2) + P(\mathbf{B}_1 \cap \mathbf{A}_2) = P(\mathbf{A}_1)P(\mathbf{B}_2|\mathbf{A}_1) + P(\mathbf{B}_1)P(\mathbf{A}_2|\mathbf{B}_1) = (1/3)(1/2) + (1/3)(1/2) = 1/3.$

$P(\mathbf{R}|\mathbf{U}) = P(\mathbf{R}_a \cap \mathbf{R}_b) = P(\mathbf{R}_a)P(\mathbf{R}_b) = (3/10)(12/20) = 36/200 = 9/50$

Step 2: $\mathbf{R} = [\mathbf{R} \cap (\mathbf{A} \cap \mathbf{B}] \cup [\mathbf{R} \cap (\mathbf{A} \cap \mathbf{C}] \cup [\mathbf{R} \cap (\mathbf{B} \cap \mathbf{C})]$

$P(\mathbf{R}) = P[\mathbf{R} \cap (\mathbf{A} \cap \mathbf{B}] + P[\mathbf{R} \cap (\mathbf{A} \cap \mathbf{C}] + P[\mathbf{R} \cap (\mathbf{B} \cap \mathbf{C})] =$

$P[(\mathbf{R}_1 \cap \mathbf{R}_2) \cap (\mathbf{A} \cap \mathbf{B}] + P[(\mathbf{R}_1 \cap \mathbf{R}_2) \cap (\mathbf{A} \cap \mathbf{C}] + P[(\mathbf{R}_1 \cap \mathbf{R}_2) \cap (\mathbf{B} \cap \mathbf{C})] =$

$P[(\mathbf{R}_1 \cap \mathbf{R}_2)|\mathbf{A} \cap \mathbf{B}] + P[(\mathbf{R}_1 \cap \mathbf{R}_2)|\mathbf{A} \cap \mathbf{C}] + P[(\mathbf{R}_1 \cap \mathbf{R}_2)|\mathbf{B} \cap \mathbf{C}] =$

$P(\mathbf{A} \cap \mathbf{B})P[(\mathbf{R}_1 \cap \mathbf{R}_2)|\mathbf{A} \cap \mathbf{B}] + P(\mathbf{A} \cap \mathbf{C})P[(\mathbf{R}_1 \cap \mathbf{R}_2)|\mathbf{A} \cap \mathbf{C}] + P(\mathbf{B} \cap \mathbf{C})P[(\mathbf{R}_1 \cap \mathbf{R}_2)|\mathbf{B} \cap \mathbf{C}] =$

$(1/3)(3/10)(12/20) + (1/3)(3/10)(5/10) + (1/3)(12/20)(5/10) = 21/100.$

Step 3: $P(\mathbf{U}|\mathbf{R}) = P(\mathbf{R}|\mathbf{U})P(\mathbf{U})/P(\mathbf{R}) = (9/50)(1/3)(100/21) = 2/7$

8.
R: The event student attended R-rated films.
V: The event student committed violents crimes.

From the statement of the problem, we find:

$P(\mathbf{V}) = 0.07$
$P(\mathbf{V}') = 1 - 0.07 = 0.93$
$P(\mathbf{R}|\mathbf{V}) = 0.95$
$P(\mathbf{R}|\mathbf{V}') = 0.15$

$P(\mathbf{V}|\mathbf{R}) = P(\mathbf{R}|\mathbf{V})P(\mathbf{V})/P(\mathbf{R}) = (0.95)(0.07)/P(\mathbf{R}) = 0.0665/P(\mathbf{R})$

$\mathbf{R} = (\mathbf{R} \cap \mathbf{V}) \cup (\mathbf{R} \cap \mathbf{V}')$

$P(\mathbf{R}) = P(\mathbf{R} \cap \mathbf{V}) + P(\mathbf{R} \cap \mathbf{V}') = P(\mathbf{V})P(\mathbf{R}|\mathbf{V}) + P(\mathbf{V}')P(\mathbf{R}|\mathbf{V}') = (0.07)(0.95) + (0.93)(0.15) = 0.206$

Therefore, $P(\mathbf{V}|\mathbf{R}) = 0.0665/P(\mathbf{R}) = 0.0665/0.206 = 0.32$

9.

$P[\mathbf{C}\cap\mathbf{B}|\mathbf{A}] = P(\mathbf{C}\cap\mathbf{B}\cap\mathbf{A})/P(\mathbf{A}) = \{P(\mathbf{A})P(\mathbf{B}|\mathbf{A})P(\mathbf{C}|\mathbf{A}\cap\mathbf{B})\}/P(\mathbf{A}) = P(\mathbf{B}|\mathbf{A})P(\mathbf{C}|\mathbf{A}\cap\mathbf{B})$

10.

Step 1: $P(\mathbf{C}|\mathbf{A}\cap\mathbf{B}) = \dfrac{P(\mathbf{A}\cap\mathbf{B}|\mathbf{C})P(\mathbf{C})}{P(\mathbf{A}\cap\mathbf{B})}$, Baye's formula.

Step 2: From problem 9, $P[\mathbf{A}\cap\mathbf{B}|\mathbf{C}] = P(\mathbf{B}|\mathbf{C})P(\mathbf{A}|\mathbf{B}\cap\mathbf{C})$

Substituting into Step 1, we have

$$P(\mathbf{C}|\mathbf{A}\cap\mathbf{B}) = \frac{P(\mathbf{A}\cap\mathbf{B}|\mathbf{C})P(\mathbf{C})}{P(\mathbf{A}\cap\mathbf{B})} = \frac{P(\mathbf{B}|\mathbf{C})P(\mathbf{A}|\mathbf{B}\cap\mathbf{C})P(\mathbf{C})}{P(\mathbf{A}\cap\mathbf{B})}$$

11.

➤ a.

R: A red marble is selected.

$\mathbf{R}_1$: A red marble is selected from table 1.

$\mathbf{R}_2$: A red marble is selected from table 2.

$\mathbf{T}_1$: Table 1 is selected.

$\mathbf{T}_2$: Table 2 is selected.

A: urn A is selected.

B: urn B is selected.

C: urn C is selected.

D: urn D is selected.

Step 1: $P(\mathbf{T}_2|\mathbf{R}) = P(\mathbf{R}|\mathbf{T}_2)\mathbf{P}(\mathbf{T}_2)/P(\mathbf{R}) = P(\mathbf{R}_2|\mathbf{T}_2)\mathbf{P}(\mathbf{T}_2)/P(\mathbf{R})$

$\mathbf{R}_2 = (\mathbf{R}_2\cap\mathbf{C})\cup(\mathbf{R}_2\cap\mathbf{D})$

$P(\mathbf{T}_2|\mathbf{R}) = P(\mathbf{R}_2|\mathbf{T}_2)\mathbf{P}(\mathbf{T}_2)/P(\mathbf{R}) = P[(\mathbf{R}_2\cap\mathbf{C})\cup(\mathbf{R}_2\cap\mathbf{D})]\ |\mathbf{T}_2)\mathbf{P}(\mathbf{T}_2)/P(\mathbf{R}) =$

$\{P[(\mathbf{R}_2\cap\mathbf{C})|\mathbf{T}_2] + P[(\mathbf{R}_2\cap\mathbf{D})]\ |\mathbf{T}_2]\}\mathbf{P}(\mathbf{T}_2)/P(\mathbf{R})$

Step 2: $P[(\mathbf{R}_2\cap\mathbf{C})|\mathbf{T}_2] = P(\mathbf{C}|\mathbf{T}_2)P(\mathbf{R}_2|\mathbf{C}\cap\mathbf{T}_2) = (1/2)(10/20) = 10/40$

$P[(\mathbf{R}_2\cap\mathbf{D})|\mathbf{T}_2] = P(\mathbf{D}|\mathbf{T}_2)P(\mathbf{R}_2|\mathbf{D}\cap\mathbf{T}_2) = (1/2)(5/20) = 5/40$

$P(\mathbf{T}_2|\mathbf{R}) = \{P[(\mathbf{R}_2\cap\mathbf{C})|\mathbf{T}_2] + P[(\mathbf{R}_2\cap\mathbf{D})]\ |\mathbf{T}_2]\}\mathbf{P}(\mathbf{T}_2)/P(\mathbf{R}) = \{10/40 + 5/40\}\mathbf{P}(\mathbf{T}_2)/P(\mathbf{R}) =$

$(15/40)\mathbf{P}(\mathbf{T}_2)/P(\mathbf{R}) = [(15/40)(1/2)]/P(\mathbf{R}) = 3/(16P(\mathbf{R}))$

Step 3: $\mathbf{R} = (\mathbf{R}\cap\mathbf{T}_1\cap\mathbf{A})\cup(\mathbf{R}\cap\mathbf{T}_1\cap\mathbf{B})\cup(\mathbf{R}\cap\mathbf{T}_2\cap\mathbf{C})\cup(\mathbf{R}\cap\mathbf{T}_2\cap\mathbf{D})$

$P(\mathbf{R}) = P(\mathbf{R}\cap\mathbf{T}_1\cap\mathbf{A}) + P(\mathbf{R}\cap\mathbf{T}_1\cap\mathbf{B}) + P(\mathbf{R}\cap\mathbf{T}_2\cap\mathbf{C}) + P(\mathbf{R}\cap\mathbf{T}_2\cap\mathbf{D}) =$

$P(\mathbf{T}_1)P(\mathbf{A}|\mathbf{T}_1)P(\mathbf{R}|\mathbf{T}_1\cap\mathbf{A})+ P(\mathbf{T}_1)P(\mathbf{B}|\mathbf{T}_1)P(\mathbf{R}|\mathbf{T}_1\cap\mathbf{B})+ P(\mathbf{T}_2)P(\mathbf{C}|\mathbf{T}_2)P(\mathbf{R}|\mathbf{T}_2\cap\mathbf{C}) +$

$P(\mathbf{T}_2)P(\mathbf{A}|\mathbf{T}_2)P(\mathbf{R}|\mathbf{T}_2\cap\mathbf{D}) =$

$(1/2)(1/2)(12/20) + (1/2)(1/2)(15/20) + (1/2)(1/2)(10/20) + (1/2)(1/2)(5/20) =$

$(1/4)(12/20 + 15/20 + 10/20 + 5/20) = 42/80 = 21/40$

Step 4: $P(\mathbf{T}_2|\mathbf{R}) = 3/(16P(\mathbf{R})) = (3/16)(40/21) = 120/336 = 5/14$

➤ b.
$P(\mathbf{C}|\mathbf{R}) = P(\mathbf{R}|\mathbf{C})P(\mathbf{C})/P(\mathbf{R})$

$P(\mathbf{R}|\mathbf{C}) = 10/20 = 1/2$

$\mathbf{C} = (\mathbf{C}\cap\mathbf{T}_1)\cup(\mathbf{C}\cap\mathbf{T}_2)$
$P(\mathbf{C}) = P(\mathbf{C}\cap\mathbf{T}_1) + P(\mathbf{C}\cap\mathbf{T}_2) = 0 + P(\mathbf{T}_2)P(\mathbf{C}|\mathbf{T}_2) = (1/2)(1/2) = 1/4$

From a. we have $P(\mathbf{R}) = 21/40$

Therefore, $P(\mathbf{C}|\mathbf{R}) = P(\mathbf{R}|\mathbf{C})P(\mathbf{C})/P(\mathbf{R}) = (1/2)(1/4)/(21/40) = (1/8)(40/21) = 5/21$

12.
L: Lung disease
S: Smokers

$P(\mathbf{S}|\mathbf{L}) = 0.68$
$P(\mathbf{L}|\mathbf{S}) = 0.15$
$P(\mathbf{L}) = 0.07$

$P(\mathbf{L}) = 0.07$

$P(\mathbf{L}|\mathbf{S}) = P(\mathbf{S}|\mathbf{L})P(\mathbf{L})/P(\mathbf{S})$

$P(\mathbf{S})P(\mathbf{L}|\mathbf{S}) = P(\mathbf{S}|\mathbf{L})P(\mathbf{L})$

$P(\mathbf{S})(0.15) = (0.68)(0.07) = 0.0476$

$P(\mathbf{S}) = (0.0476)/(0.15) \approx 0.32$

13.
D: diamond drawn from deck B.
$\mathbf{D}_1$ first card drawn from deck A is a diamond.
$\mathbf{D}_2$ second card drawn from deck A is a diamond.
$\mathbf{C}_1$ first card drawn from deck A is a club.
$\mathbf{C}_2$ second card drawn from deck A is a club.

➤ a.
$\mathbf{D}_1 \cup \mathbf{D}_2$: The event at least 1 diamond is drawn from deck A.

$(\mathbf{D}_1 \cup \mathbf{D}_2)' = \mathbf{D}_1' \cap \mathbf{D}_2'$: No diamond is drawn from deck A.

$P(\mathbf{D}_1 \cup \mathbf{D}_2 | \mathbf{D}) = 1 - P(\mathbf{D}_1' \cap \mathbf{D}_2' | \mathbf{D})$

Using Baye's we have

$P(\mathbf{D}_1' \cap \mathbf{D}_2' | \mathbf{D}) = P(\mathbf{D} | \mathbf{D}_1' \cap \mathbf{D}_2')P(\mathbf{D}_1' \cap \mathbf{D}_2')/P(\mathbf{D}) = [(13/54)(39/52)(38/51)]/P(\mathbf{D}) =$
$19266/[143208P(\mathbf{D})]$

$\mathbf{D} = [(\mathbf{D}_1' \cap \mathbf{D}_2') \cap \mathbf{D}] \cup [(\mathbf{D}_1 \cap \mathbf{D}_2') \cap \mathbf{D}] \cup [(\mathbf{D}_1' \cap \mathbf{D}_2) \cap \mathbf{D}] \cup [(\mathbf{D}_1 \cap \mathbf{D}_2) \cap \mathbf{D}]$

$P(\mathbf{D}) = P[(\mathbf{D}_1' \cap \mathbf{D}_2') \cap \mathbf{D}] + P[(\mathbf{D}_1 \cap \mathbf{D}_2') \cap \mathbf{D}] + P[(\mathbf{D}_1' \cap \mathbf{D}_2) \cap \mathbf{D}] + P[(\mathbf{D}_1 \cap \mathbf{D}_2) \cap \mathbf{D}] =$

$P(\mathbf{D}_1' \cap \mathbf{D}_2')P[\mathbf{D} | (\mathbf{D}_1' \cap \mathbf{D}_2')] + P(\mathbf{D}_1 \cap \mathbf{D}_2')P[\mathbf{D} | (\mathbf{D}_1 \cap \mathbf{D}_2')] + P(\mathbf{D}_1' \cap \mathbf{D}_2)P[\mathbf{D} | (\mathbf{D}_1' \cap \mathbf{D}_2)] +$

$P(\mathbf{D}_1 \cap \mathbf{D}_2)P[|\mathbf{D} | (\mathbf{D}_1 \cap \mathbf{D}_2)] = (39/52)(38/51)(13/54) + (13/52)(39/51)(14/54) + (39/52)(13/51)(14/54)$

$+ (13/52)(12/51)(15/54) = 19266/143208 + 7098/143208 + 7098/142308 + 2340/143208 =$

$71586/143208$

$P(\mathbf{D}_1' \cap \mathbf{D}_2' | \mathbf{D}) = 19266/[143208P(\mathbf{D})] = 19266/71586$

$P(\mathbf{D}_1 \cup \mathbf{D}_2 | \mathbf{D}) = 1 - P(\mathbf{D}_1' \cap \mathbf{D}_2' | \mathbf{D}) = 1 - 19266/71586 = 52320/71586$

➤ b.
$\mathbf{E} = (\mathbf{D}_1 \cap \mathbf{C}_2) \cup (\mathbf{C}_1 \cap \mathbf{D}_2)$

$P(\mathbf{E} | \mathbf{D}) = P((\mathbf{D}_1 \cap \mathbf{C}_2) \cup (\mathbf{C}_1 \cap \mathbf{D}_2) | \mathbf{D}) = P(\mathbf{D}_1 \cap \mathbf{C}_2 | \mathbf{D}) + P(\mathbf{C}_1 \cap \mathbf{D}_2 | \mathbf{D})$

$P(\mathbf{D}_1 \cap \mathbf{C}_2 | \mathbf{D}) = P(\mathbf{D} | \mathbf{D}_1 \cap \mathbf{C}_2)P(\mathbf{D}_1 \cap \mathbf{C}_2)/P(\mathbf{D}) = (14/54)(13/52)(13/51)(143208/71586) =$

$(2366/143208)(143208/71586) = 2366/71586$

Using the same argument:

$P(\mathbf{C}_1 \cap \mathbf{D}_2 | \mathbf{D}) = 21366/71586$

Therefore,

$P(\mathbf{E}|\mathbf{D}) = P(\mathbf{D}_1 \cap \mathbf{C}_2|\mathbf{D}) + P(\mathbf{C}_1 \cap \mathbf{D}_2|\mathbf{D}) = 2(21366/71586) = 21366/35793 = 7122/11931$

14.
We assume $\mathbf{S} = \mathbf{A}_1 \cup \mathbf{A}_2 \cup \ldots \cup \mathbf{A}_n$ and $\mathbf{A}_j \cap \mathbf{A}_k = \phi$ for all $j \neq k$.

From the definition of Baye's formula:

$$P(\mathbf{A}|\mathbf{B}) = \frac{P(\mathbf{B}|\mathbf{A})P(\mathbf{A})}{P(\mathbf{B})}.$$

$\mathbf{B} = \mathbf{B} \cap \mathbf{S} = \mathbf{B} \cap (\mathbf{A}_1 \cup \mathbf{A}_2 \cup \ldots \cup \mathbf{A}_n) = (\mathbf{B} \cap \mathbf{A}_1) \cup (\mathbf{B} \cap \mathbf{A}_2) \cup \ldots \cup (\mathbf{B} \cap \mathbf{A}_n)$

$P(\mathbf{B}) = P[(\mathbf{B} \cap \mathbf{A}_1) \cup (\mathbf{B} \cap \mathbf{A}_2) \cup \ldots \cup (\mathbf{B} \cap \mathbf{A}_n)] = P(\mathbf{B} \cap \mathbf{A}_1) + P(\mathbf{B} \cap \mathbf{A}_2) + \ldots + P(\mathbf{B} \cap \mathbf{A}_n) =$

$= P(\mathbf{A}_1)P(\mathbf{B}|\mathbf{A}_1) + P(\mathbf{A}_2)P(\mathbf{B}|\mathbf{A}_2) + \ldots + P(\mathbf{A}_n)P(\mathbf{B}|\mathbf{A}_n)$

Substitute into the denominator completes the formula.

15.

$\mathbf{R}_k$: k th red ball drawn from urn A.

$\mathbf{B}_k$**:** th red ball drawn from urn B.

$\mathbf{R}$: the event that only 1 red ball was drawn from urn A.

$\mathbf{R} = (\mathbf{R}_1 \cap \mathbf{R}_2') \cup (\mathbf{R}_1' \cap \mathbf{R}_2)$

$P(\mathbf{R}|\mathbf{B}_1 \cap \mathbf{B}_2) = P\{(\mathbf{R}_1 \cap \mathbf{R}_2') \cup (\mathbf{R}_1' \cap \mathbf{R}_2)|\mathbf{B}_1 \cap \mathbf{B}_2\} = P\{(\mathbf{R}_1 \cap \mathbf{R}_2'|\mathbf{B}_1 \cap \mathbf{B}_2)\} + P\{(\mathbf{R}_1' \cap \mathbf{R}_2)|\mathbf{B}_1 \cap \mathbf{B}_2\}$

$P(\mathbf{B}_1 \cap \mathbf{B}_2|\mathbf{R}_1 \cap \mathbf{R}_2')P(\mathbf{R}_1 \cap \mathbf{R}_2')/P(\mathbf{B}_1 \cap \mathbf{B}_2) + P\{(\mathbf{B}_1 \cap \mathbf{B}_2)|\mathbf{R}_1' \cap \mathbf{R}_2\}P(\mathbf{R}_1' \cap \mathbf{R}_2)/ P(\mathbf{B}_1 \cap \mathbf{B}_2) =$

$(4/10)(3/9)(10/15)(5/14)/P(\mathbf{B}_1 \cap \mathbf{B}_2) + (4/10)(3/9)(5/15)(10/14)/P(\mathbf{B}_1 \cap \mathbf{B}_2) = [1200/18900]/P(\mathbf{B}_1 \cap \mathbf{B}_2)$

$\mathbf{B}_1 \cap \mathbf{B}_2 = [(\mathbf{B}_1 \cap \mathbf{B}_2) \cap (\mathbf{R}_1 \cap \mathbf{R}_2)] \cup [(\mathbf{B}_1 \cap \mathbf{B}_2) \cap (\mathbf{R}_1' \cap \mathbf{R}_2)] \cup [(\mathbf{B}_1 \cap \mathbf{B}_2) \cap (\mathbf{R}_1 \cap \mathbf{R}_2')] \cup [(\mathbf{B}_1 \cap \mathbf{B}_2) \cap (\mathbf{R}_1' \cap \mathbf{R}_2')]$

$P(\mathbf{B}_1 \cap \mathbf{B}_2) = P[(\mathbf{B}_1 \cap \mathbf{B}_2) \cap (\mathbf{R}_1 \cap \mathbf{R}_2)] + P[(\mathbf{B}_1 \cap \mathbf{B}_2) \cap (\mathbf{R}_1' \cap \mathbf{R}_2)] +$

$P[(\mathbf{B}_1 \cap \mathbf{B}_2) \cap (\mathbf{R}_1 \cap \mathbf{R}_2')] + P[(\mathbf{B}_1 \cap \mathbf{B}_2) \cap (\mathbf{R}_1' \cap \mathbf{R}_2')] = (3/10)(2/9) + (4/10)(3/9) + (4/10)(3/9) + (5/10)(4/9)$

$= 50/90$

$P(\mathbf{R}|\mathbf{B}_1 \cap \mathbf{B}_2) = [1200/18900]/(50/90) = (1200/18900)(90/50) = 4/35$

16.

$\mathbf{R}_k$: k th red ball drawn from urn A.

$\mathbf{B}_k$**:** th red ball drawn from urn B.

$\mathbf{B}$: at least one black ball was drawn from urn B.

$\mathbf{R}$: two red balls were drawn from urn A.

$P(\mathbf{R} | \mathbf{B}) = P(\mathbf{B} | \mathbf{R})P(\mathbf{R})/P(\mathbf{B})$

$P(\mathbf{B} \mid \mathbf{R}) = P(\mathbf{B}_1 \cup \mathbf{B}_2 \mid \mathbf{R}) = P(\mathbf{B}_1 \mid \mathbf{R}) + P(\mathbf{B}_2 \mid \mathbf{R}) - P(\mathbf{B}_1 \cap \mathbf{B}_2 \mid \mathbf{R}) = 3/10 + 3/10 - (3/10)(2/9) = 48/90$

$P(\mathbf{R}) = (10/15)(9/14) = 90/210$

$P(\mathbf{R} \mid \mathbf{B}) = P(\mathbf{B} \mid \mathbf{R})P(\mathbf{R})P(\mathbf{R} \mid \mathbf{B}) = P(\mathbf{B} \mid \mathbf{R})P(\mathbf{R})/P(\mathbf{B})=$

$(48/90)\ (90/210)/P(\mathbf{R} \mid \mathbf{B}) = P(\mathbf{B} \mid \mathbf{R})P(\mathbf{R})/P(\mathbf{B}) = (24/105)/P(\mathbf{B})$

$\mathbf{B} = \mathbf{B} \cap (\mathbf{R}_1 \cap \mathbf{R}_2) \cup \mathbf{B} \cap (\mathbf{R}_1' \cap \mathbf{R}_2) \cup \mathbf{B} \cap (\mathbf{R}_1 \cap \mathbf{R}_2') \cup \mathbf{B} \cap (\mathbf{R}_1' \cap \mathbf{R}_2')$

$P(\mathbf{B}) = P[\mathbf{B} \cap (\mathbf{R}_1 \cap \mathbf{R}_2)] + P[\mathbf{B} \cap (\mathbf{R}_1' \cap \mathbf{R}_2)] + P[\mathbf{B} \cap (\mathbf{R}_1 \cap \mathbf{R}_2')] + P[\mathbf{B} \cap (\mathbf{R}_1' \cap \mathbf{R}_2')]$

$P[\mathbf{B} \cap (\mathbf{R}_1 \cap \mathbf{R}_2)] = P[(\mathbf{B}_1 \cup \mathbf{B}_2) \mid (\mathbf{R}_1 \cap \mathbf{R}_2)]P(\mathbf{R}_1 \cap \mathbf{R}_2)] =$

$P[(\mathbf{B}_1 \mid (\mathbf{R}_1 \cap \mathbf{R}_2)]P(\mathbf{R}_1 \cap \mathbf{R}_2)] + P[(\mathbf{B}_2 \mid (\mathbf{R}_1 \cap \mathbf{R}_2)]P(\mathbf{R}_1 \cap \mathbf{R}_2)] - P[(\mathbf{B}_1 \cap \mathbf{B}_2 \mid (\mathbf{R}_1 \cap \mathbf{R}_2)]P(\mathbf{R}_1 \cap \mathbf{R}_2)] =$

$(3/10)(10/15)(9/14) + (3/10)(10/15)(9/14) - (3/10)(2/9)(10/15)(9/14) = 4860/18900 - 540/18900$

$4320/18900$

$P[\mathbf{B} \cap (\mathbf{R}_1' \cap \mathbf{R}_2)] =$

$P[(\mathbf{B}_1 \mid (\mathbf{R}_1' \cap \mathbf{R}_2)]P(\mathbf{R}_1' \cap \mathbf{R}_2)] + P[(\mathbf{B}_2 \mid (\mathbf{R}_1' \cap \mathbf{R}_2)]P(\mathbf{R}_1' \cap \mathbf{R}_2)] - P[(\mathbf{B}_1 \cap \mathbf{B}_2 \mid (\mathbf{R}_1' \cap \mathbf{R}_2)]P(\mathbf{R}_1' \cap \mathbf{R}_2)] =$

$(4/10)(5/15)(10/14) + (4/10)(5/15)(10/14) - (4/10)(3/9)(5/15)(10/14) = 3600/18900 - 600/18900$

$= 3000/18900$

$P[\mathbf{B} \cap (\mathbf{R}_1 \cap \mathbf{R}_2')] = 3000/18900$

$P[\mathbf{B} \cap (\mathbf{R}_1' \cap \mathbf{R}_2')] =$

$P[(\mathbf{B}_1 \mid (\mathbf{R}_1' \cap \mathbf{R}_2')]P(\mathbf{R}_1' \cap \mathbf{R}_2')] + P[(\mathbf{B}_2 \mid (\mathbf{R}_1' \cap \mathbf{R}_2')]P(\mathbf{R}_1' \cap \mathbf{R}_2')] - P[(\mathbf{B}_1 \cap \mathbf{B}_2 \mid (\mathbf{R}_1' \cap \mathbf{R}_2')]P(\mathbf{R}_1' \cap \mathbf{R}_2')] =$

$(5/10)(5/15)(4/14) + (5/10)(5/15)(4/14) - (5/10)(4/10)(5/15)(4/14) = 1800/18900 - 400/18900$

$1400/18900$

$P(\mathbf{B}) = P[\mathbf{B} \cap (\mathbf{R}_1 \cap \mathbf{R}_2)] + P[\mathbf{B} \cap (\mathbf{R}_1' \cap \mathbf{R}_2)] + P[\mathbf{B} \cap (\mathbf{R}_1 \cap \mathbf{R}_2')] + P[\mathbf{B} \cap (\mathbf{R}_1' \cap \mathbf{R}_2')] =$

$4320/18900 + 3000/18900 + 3000/18900 + 1400/18900 = 11720/18900$

$P(\mathbf{R} \mid \mathbf{B}) = P(\mathbf{B} \mid \mathbf{R})P(\mathbf{R})/P(\mathbf{B}) = (24/105)(18900/11720) = 4536/12306$