PROBABILITY THEORY
Lesson 10
Probability Sample Space

10.1- What is a Probability Space?

10.1 - Problem 1:
➤(a).
Step 1: The sample space **S** is the number of students that will complete the class:
S = {0,1,2,3,...50}.

Step 2: We wish to find the probability of the event **E** = *At the end of the semester, at least 49 students will finish the class.*

Step 3: **E** = {49,50}.

Step 4: **E**' = *At most 48 students will finish the class*

Step 5: From the problem, we know P(**E**′) = 0.72.

Step 6: P(**E**) = 1 - P(**E'**) = 1 - 0.72 = 0.28.

➤(b).
Since probability of an event occurring can be interpreted as the percent of chance, we can write the event **E** as:

There is a 28% chance that at the end of the semester Ms. Romano's Latin class will have at least 49 students remaining.

10.1 - Problem 2:
➤(a).
Step 1: Since we have no idea as to the number of defective computer chips received per shipment, we can assume the sample space of defective chips is **S** = {0,1,2,3,4,5,...}.

Step 2: From the problem we can write the following events:

A = *At least 10 chips are defective.* = {10,11,12,...}

Step 3: **B** = *That between 5 and 20 chips are defective.* = {5,6,7,8,9,...,20}.

Step 4: **C** = *That between 10 and 20 chips are defective.* = {10,11,..,20}.

Step 5: **E** = *That the box contains more than 4 defective chips* = {5,6,7,8,9,10,...}.

Step 6: We have to write **E** an a expression of the sets **A**,**B**,**C:**

A∪**B** = {10,11,12,...}∪{5,6,7,8,9,...,20} = {5,6,7,8,...,} = **E**.

Step 7: P(**E**) = P(**A**∪**B**) = P(**A**) + P(**B**) - P(**A**∩**B**) = 0.15 + 0.30 - P(**A**∩**B**).

Step 8: **A**∩**B** = {10,11,12,...}∪{5,6,7,8,9,...,20} = {10,11,...,20} = **C**

Step 9: P(**A**∩**B**) = P(**C**) = 0.05.

Step 10: P(**E**) = P(**A**∪**B**) = P(**A**) + P(**B**) - P(**A**∩**B**) = 0.15 + 0.30 - 0.05 = 0.40.

➤(b).
Since probability of an event occurring can be interpreted as the percent of chance, we can write the event **E** as: There is a 40% chance that the box contains more than 4 defective chips.

10.1 - Problem 3:
➤(a). British History.
A: The event she takes American history.
B: The event she takes British history.
G: The event she takes Greek history.

S = A∪B∪G

P(**S**) = P(**A**) + P(**B**) + P(**G**) = 0.53 + P(**B**) + .25 = 1

Solving this equation gives P(**B**) = 1 - 0.53 - 0.25 = 1 - 0.78 = 0.22

➤(b).
Step 1: **E** = *she takes British History or Ancient Greek History.*

Step 2: **E = B∪G**

Step 3: P(**E**) = P(**B**) + P(**G**) = 0.22 + 0.25 = 0.47.

10.1 - Problem 4:
Step 1: **S** = *All possible number of errors in the book* = {0,1,3,...}.

Step 2: **A** = *The book has at least 50 errors* = {50,51,52,...}.

B = *The book has less than 60 errors* = {0,1,2,3,...59}.

E = *The American history books contains between 50 and 59 errors* = { 50,51,52,...,59}.

Step 3: **E** = { 50,51,52,...,59} = {50,51,52,...}∩{0,1,2,3,...59} = **A**∩**B.**

Step 4: P(**E**) = P(**A**∩**B**) = P(**A**) + P(**B**) - P(**A**∪**B**)

Step 5: **A**∪**B** = {50,51,52,...}∪{0,1,2,3,...59} = {0,1,2,3,...,} = **S**

Step 6: P(**E**) = P(**A**) + P(**B**) - P(**A**∪**B**) = P(**A**) + P(**B**) - P(**S**) = P(**A**) + P(**B**) - 1

Step 7: P(**A**) = 0.35

P(**B**) = 0.70

Step 8: P(**E**) = P(**A**) + P(**B**) - 1 = 0.35 + 0.70 - 1 = 0.05

10.1 - Problem 5:
➤(a).
Step 1: **A** = {100,101,...,160}

B = {150,151,152,...,170}

C = {160,161,...,170}.

E = {161,162,...160}.

Step 2: **E** = {161,162,...,170} = {100,101,...,160}′ = **A**′.

Step 3: P(**E**) = P(**A**') = 1 - P(**A**) = 1 - 0.90 = 0.10.

➤(b).
Step 1: **E** = {100,101,102,...,149}

Step 2: **E**' = {100,101,102,...,149}′ = {150,151,...,170} = **B.**

Step 3: P(**E**) = 1 - P(**E**′) = 1 - P(**B**) = 1 - 0.85 = 0.15.

➤(c).
Step 1: **E** = {150,152,...,159}

Step 2: **E** = {150,152,...,159} = {150,151,152,...,170}∩{160,161,...,170}′ = **B**∩**C**′

Step 3: P(**E**) = P(**B**∩**C**′) = P(**B**) + P(**C**′) - P(**B**∪**C**′) = 0.85 + 0.75 - P(**B**∪**C**′)

Step 5: **B**∪**C**′ = {150,151,152,..,170}∪{160,161,...,170}' = {150,151,152,..,170}∪{100,101,...,159} = **S**

Step 6: P(**E**) = P(**B**∩**C**′) = P(**B**) + P(**C**′) - P(**B**∪**C**′) = 0.85 + 0.75 - P(**S**) = 0.85 + 0.75 - 1 = 0.60

➤(d).
Step 1: **E** = {160} = {100,101,...,160}∩{160,161,...,170} = **A**∩**C**

Step 2: **E** = **A**∩**C**

Step 3: P(**E**) = P(**A**∩**C**) = P(**A**) + P(**C**) - P(**A**∪**C**) = 0.90 + 0.25 - P(**A**∪**C**)

Step 4: **A**∪**C** = {100,101,...,160}∪{160,161,...,170} = **S**

Step 5: P(**E**) = P(**A**∩**C**) = P(**A**) + P(**C**) - P(**A**∪**C**) = 90 + 0.25 - P(**S**) =90 + 0.25 - 1 = 0.15

➤(e).
Step 1: The event *less than 150 mph:* **B**′

The event *between 160 and 170 mph:* **C**

Step 2: The event *less than 150 mph* **or** *between 160 and 170 mph*: **B**′∪**C.**

Step 3: P(**B**′) = 1 - P(**B**) = 1 - 0.85 = 0.15, P(**C**) = 0.25.

Since **B**′∩**C** = ϕ, P(**B**′∪**C**) = P(**B**′) + P(**C**) **=** 0.15 + 0.25 = 0.40.

10.2 - Defining the Probability of Events when Elements of the Sample Space are equal likely to Occur.

10.2 - Problem 1:
Step 1: **S** = *Fifty-two possible cards.*

Step 2: **E** = {ace, two,...,ten, jack, queen, king of diamonds}.

Step 3: #**S** = 52

Step 4: #**E** = 13.

Step 5 P(**E**) = 13/52.

10.2 - Problem 2:

➤(a).

Step 1: **S** = {(h,h,h),(h,h,t),(h,t,h),(t,h,h),(t,t,h),(t,h,t),(h,t,t),(t,t,t)}

Step 2: **E** = {(h,h,t),(h,t,h),(t,h,h)}

Step 3: #**S** = 8

#**E** = 3

Step 4: P(**E**) = 3/8

➤(b).

Step 1: **E** = *at least one tail occurs*

E′ = *no tail occurs* = {(h,h,h)}

Step 2: #**E**′ = 1

Step 3: P(**E**′) = 1/8.

Step 4: P(**E**) = 1 - P(**E**′) = 1 - 1/8 = 7/8.

➤(c).

Step 1: **E** = *all sides are the same* = {(h,h,h),(t,t,t)}.

Step 2: #**E** = 2

P(**E**) = 2/8

10.2 - Problem 3:

➤(a).

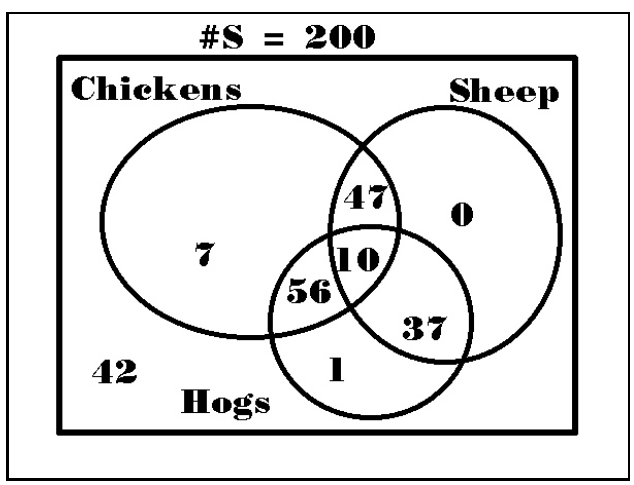
C: the farmers that raise chickens

H: the farmers that raise hogs

K: the farmers that raise sheep

Step 1: **E** = *the event that the farmer selected only raises sheep* = **K**∩(**C**∪**H**)′

Step 2: From the venn diagram, we see that there are no farmers that only raise sheep.



Step 3: Therefore, #**E** = 0

Step 4: Since #**S =** 200, then

P(**E**) = 0/200 = 0

➤(b).
Step 1: **E =** *the event that the farmer selected raises sheep and hogs but not chickens* = $\mathbf{K}\cap\mathbf{H}\cap\mathbf{C}'$

Step 2: From the Venn diagram, $\#\mathbf{K}\cap\mathbf{H}\cap\mathbf{C}' = 37$.

Step 3: P(**E**) = 37/200.

➤(c).
Step 1: **E** = *only raises one of these animals* = $(\mathbf{K}\cap\mathbf{H}'\cap\mathbf{C}')\cup(\mathbf{K}'\cap\mathbf{H}\cap\mathbf{C}')\cup(\mathbf{K}'\cap\mathbf{H}'\cap\mathbf{C})$

Step 2: From the Venn diagram,

$\#\,\mathbf{E} = \#[(\mathbf{K}\cap\mathbf{H}'\cap\mathbf{C}')\cup(\mathbf{K}'\cap\mathbf{H}\cap\mathbf{C}')\cup(\mathbf{K}'\cap\mathbf{H}'\cap\mathbf{C})] = \#(\mathbf{K}\cap\mathbf{H}'\cap\mathbf{C}') + \#(\mathbf{K}'\cap\mathbf{H}\cap\mathbf{C}') + \#(\mathbf{K}'\cap\mathbf{H}'\cap\mathbf{C})]$
$= 0 + 1 + 7 = 8$.

Step 3: P(**E**) = 8/200.

➤(d).
Step 1: **E** = *raises sheep or hogs but not chickens* = $(\mathbf{K}\cup\mathbf{H})\cap\mathbf{C}'$

Step 2: From the Venn diagram, $\#(\mathbf{K}\cup\mathbf{H})\cap\mathbf{C}' = 38$.

Step 3: Therefore, P(**E**) = 38/200.

➤(e).
Step 1: **E** = *does not raise any of these animals* = $\mathbf{K}'\cap\mathbf{H}'\cap\mathbf{C}' = (\mathbf{K}\cup\mathbf{H}\cup\mathbf{C})'$

Step 2: From the Venn diagram, $\#\mathbf{E} = \#(\mathbf{K}\cup\mathbf{H}\cup\mathbf{C})' = 200 - (7 + 47 + 10 + 56 + 37 + 1) = 42$.

Step 3: Therefore, P(**E**) = 42/200.

10.2 - Problem 4:

D: The event that a diamond is drawn.
F: The event that a face is drawn.
E: The event that a diamond or face card drawn.
$\mathbf{E} = \mathbf{D}\cup\mathbf{F}$

P(**D**) = 13/52

P(**F**) = 12/52

P(**E**) = P(**D**∪**F**) = P(**D**) + P(**F**) - P(**D**∩**K**) = 13/52 + 12/52 - 3/52 = 22/52.

Supplementary Problems

1.
Step 1: For each spin there are 38 possible numbers. Since there are three spins, we conclude **#S** = 38x38x38 = 54,872.

Step 2: **E** = *00 appears all three times.*

Step 3: **E** = {00,00,00}

Step 4: Since #**E** = 1, P(**E**) = 1/54,872.

2.
Step 1: **E** = *all the numbers are the same*

Step 2*:* **E** = {(0,0,0),(00,00,00),(1,1,1),(2,2,2),...,(37,37,37),(38,38,38)}

Step 3: #**E** = 38.

Step 4: P(**E**) = 38/54,872

3.
Step 1: Let x represent any of the numbers other than 5.

Step 2: **E** = *only two fives appear.*

Step 3:

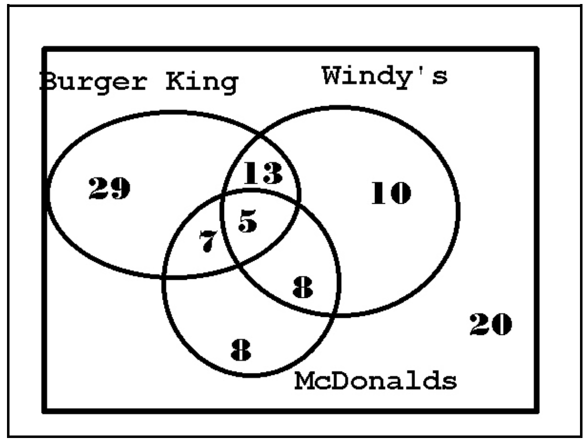
E = {(5,5,0),(5,5,00),(5,5,1),(5,5,2),(5,5,3),(5,5,4),(5,5,6),...,(5,5,38),
(5,0,5),(5,00,5),(5,1,5),(5,2,5),(5,3,5),(5,4,5),(5,6,5),...,(5,38,5),
(0,5,5),(00,5,5),(1,5,5),(2,5,5),(3,5,5),(4,5,5),(6,5,5),...,(38,5,5)

Step 4: There are 37 columns and 3 rows. Therefore, **#E** = 3x37 = 111.

Step 5: P(**E**) = 111/54,872.

4.

Step 1: #**S =** 29 + 7+ 5 + 13 + 8 + 8+ 10 + 20 = 100.



Step 2: **M** = *teenager likes McDonald's*

W = *teenager likes* Wendy's

B = *teenager likes* Burger King

Step 2: **E** = *the teenager likes McDonald's but does not like Burger King* = **M**∩**B**′

Step 3: From the Venn diagram, we see #**E** = #(**M**∩**B**′) = 8 + 8 = 16.

Step 4: P(**E**) = 16/100.

5.

Step 1: **E** = *that the teenager likes McDonald's or does not like Burger King.*

Step 2: **E** = **M**∪**B**′

Step 3: **E'** = (**M**∪**B**′)′ **= M**′∩**B**

Step 4: From the Venn diagram we see, #**E** = #(**M**′∩**B**) = 29 + 13 = 42.

Step 5: #**E** = #**S** - **#E**′ = 100 - 42 = 58.

Step 6: P(**E**) = 58/100.

6.

Step 1: Since we are given that the teenager likes Burger King and Wendy's, we can change **S** to a new sample space **S** = **B**∩**W** where **#S** = #(**B**∩**W**) **=** 5 + 13 = 18.

Step 2: **E** = *the teenager also likes McDonald* = **M**∩(**B**∩**W**).

Step 3: From the Venn diagram #**E =** 5.

Step 4: P(**E**) = 5/18.

7.

Step 1: Since we are given that the teenager likes Burger King or Wendy's, we can change **S** to a new sample space **S** = **B**∪**W** where **#S** = #(**B**∪**W**) = 72 .

Step 2: **E** = *the teenager does not likes McDonald* = $\mathbf{M}'\cap(\mathbf{B}\cup\mathbf{W})$

Step 3: From the Venn diagram #**E** = #[$\mathbf{M}'\cap(\mathbf{B}\cup\mathbf{W})$] = 29 + 13 + 10 = 52.

Step 4: P(**E**) = 52/72 = 13/18.

A die is tossed twice. Find the probability

8.
Step 1: **S** = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

#**S** = 36

Step 2: **E** = *that the sum of the numbers is either greater than 10 or equal to four.*

Step 3: **A** = *the sum of the numbers is either greater than 10*

A = {(5,6),(6,5),(6,6)}

#**A** = 3

B = *the sum of the numbers is equal to four.*

B = {(1,3),(2,2),(3,1)}

#**B** = 3

Step 4: **E** = $\mathbf{A}\cup\mathbf{B}$

Step 5: P(**E**) = P($\mathbf{A}\cup\mathbf{B}$) = P(**A**) + P(**B**) = 3/36 + 3/36 = 6/36.

9.
Step 1: **E** = *the sum of the numbers is between 2 and 4 inclusive*

E = {(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)}

#**E** = 6.

Step 2: P(**E**) = 6/36

10.
Step 1: **S** = *all the members of the House of Representatives that voted on the amendment.*

#**S** = 432

Step 2: **E** = *a randomly selected representative Congressman is selected.*

#**E** = 1

Step 3: P(**E**) = 1/432

11.
Step 1: **E** = *that a selected representative is both a Democrat and voted in favor of the amendment*

Step 2: **A** = *that a selected representative is a Democrat*

B = *that a selected representative voted in favor of the amendment*

Step 3: **E** = **A**∩**B**

From the table, we see that there were 153 Democrats that voted for the amendment.

#**E** = 153.

Step 4: P(**E**) = 153/432

12.
Step 1: Since we are given that a representative voted for the amendment, we are restricted the underlined column of our table:

	YEA	**NAY**	**DID NOT VOTE**	**Total**
Democrat	153	83	19	255
Republican	169	0	8	177
Total	322	83	27	432

Step 2: Our new sample space **S** consists of only those representatives who voted *Yea* on the amendment. Therefore, #S = 322.

Step 3: $\mathbf{E}$ = *that he/she is a Democrat.*

$\#\mathbf{E} = 153$.

Step 4: Therefore, $P(\mathbf{E}) = 153/322$.

13.

➤ a.

$P(\mathbf{A}\cup\mathbf{B}\cup\mathbf{C}) = P[\mathbf{A}\cup(\mathbf{B}\cup\mathbf{C})] = P(\mathbf{A}) + P(\mathbf{B}\cup\mathbf{C}) - P[\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C})] =$

$P(\mathbf{A}) + P(\mathbf{B}\cup\mathbf{C}) - P[(\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{C})] = P(\mathbf{A}) + P(\mathbf{B}) + P(\mathbf{C}) - P(\mathbf{B}\cap\mathbf{C}) - P[(\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{C})]$

$= P(\mathbf{A}) + P(\mathbf{B}) + P(\mathbf{C}) - P(\mathbf{B}\cap\mathbf{C}) - \{P(\mathbf{A}\cap\mathbf{B}) + P(\mathbf{A}\cap\mathbf{C}) - P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{A}\cap\mathbf{C})\} =$

$P(\mathbf{A}) + P(\mathbf{B}) + P(\mathbf{C}) - P(\mathbf{B}\cap\mathbf{C}) - P(\mathbf{A}\cap\mathbf{B}) - P(\mathbf{A}\cap\mathbf{C}) + P(\mathbf{A}\cap\mathbf{B}\cap C)\}$

➤b.

$\mathbf{S}: = \{1,2,3,4,5,6,7,8,9,10\}$

$\mathbf{A} = \{1,3,5,7,9\}$

$\mathbf{B} = \{5,6,7,8,9\}$

$\mathbf{C} = \{3,4,5,6,7\}$

$\mathbf{E} = \mathbf{A}\cup\mathbf{B}\cup\mathbf{C}$

$\mathbf{A}\cap\mathbf{B} = \{5,7,9\}$

$\mathbf{A}\cap\mathbf{C} = \{3,5,7\}$

$\mathbf{B}\cap\mathbf{C} = \{5,6,7\}$

$\mathbf{A}\cap\mathbf{B}\cap\mathbf{C} = \{5,7\}$

$\mathbf{P(E)}= P(\mathbf{A}\cup\mathbf{B}\cup\mathbf{C})= P(\mathbf{A}) + P(\mathbf{B}) + P(\mathbf{C}) - P(\mathbf{A}\cap\mathbf{B}) - P(\mathbf{A}\cap\mathbf{C}) - P(\mathbf{B}\cap\mathbf{C}) + P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C})) =$

$= 5/10 + 5/10 + 5/10 - 3/10 - 3/10 - 3/10 + 2/10 = 8/10$.

$4/52 + 12/52 + 13/52 - 3/52 - 4/52 - 1/52 + 1/52 = 22/52$.

14.

$P(\mathbf{A}_1\cup\mathbf{A}_2\cup\mathbf{A}_3\cup\ldots\cup\mathbf{A}_n) = P[\mathbf{A}_1\cup(\mathbf{A}_2\cup\mathbf{A}_3\cup\ldots\cup\mathbf{A}_n)] =$

$P(\mathbf{A}_1)+ P(\mathbf{A}_2\cup\mathbf{A}_3\cup\ldots\cup\mathbf{A}_n) - P[\mathbf{A}_1\cap(\mathbf{A}_2\cup\mathbf{A}_3\cup\ldots\cup\mathbf{A}_n)] \leq P(\mathbf{A}_1) + P(\mathbf{A}_2\cup\mathbf{A}_3\cup\ldots\cup\mathbf{A}_n)$

$P(\mathbf{A}_1\cup\mathbf{A}_2\cup\mathbf{A}_3\cup\ldots\cup\mathbf{A}_n) \leq P(\mathbf{A}_1) + P(\mathbf{A}_2\cup\mathbf{A}_3\cup\ldots\cup\mathbf{A}_n)$

$P(\mathbf{A_2}\cup\mathbf{A}_3\cup...\cup\mathbf{A}_n) = P[\mathbf{A_2}\cup(\mathbf{A_3}\cup\mathbf{A}_4\cup...\cup\mathbf{A}_n)] =$

$P(\mathbf{A_2}) + P(\mathbf{A_3}\cup\mathbf{A}_4\cup...\cup\mathbf{A}_n) - P[\mathbf{A_2}\cap(\mathbf{A_3}\cup\mathbf{A}_4\cup...\cup\mathbf{A}_n)] \le P(\mathbf{A_2}) + P(\mathbf{A}_3\cup...\cup\mathbf{A}_n)$

$P(\mathbf{A_2}\cup\mathbf{A}_3\cup...\cup\mathbf{A}_n) \le P(\mathbf{A_2}) + P(\mathbf{A}_3\cup...\cup\mathbf{A}_n)$

$P(\mathbf{A_1}\cup\mathbf{A_2}\cup\mathbf{A}_3\cup...\cup\mathbf{A_n}) \le P(\mathbf{A_1}) + P(\mathbf{A_2}\cup\mathbf{A}_3\cup...\cup\mathbf{A}_n) \le P(\mathbf{A_1}) + P(\mathbf{A_2}) + P(\mathbf{A}_3\cup...\cup\mathbf{A}_n)$

Continuing in this manner, we have

$P(\mathbf{A_1}\cup\mathbf{A_2}\cup\mathbf{A}_3\cup...\cup\mathbf{A_n}) \le P(\mathbf{A_1}) + P(\mathbf{A_2}) + \mathbf{P}(\mathbf{A_3})... + P(\mathbf{A_n})$

15.
$P[(\mathbf{A}\cup\mathbf{B})'] = 1 - P(\mathbf{A}\cup\mathbf{B}) = 1 - 0.65 = 0.35$

$P(\mathbf{A}\cap\mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A}\cup\mathbf{B}) = 0.60 + 0.5 - 0.65 = 0.45$

$P(\mathbf{A}\cap\mathbf{B}') = P(\mathbf{A}) - P(\mathbf{A}\cap\mathbf{B}) = 0.60 - 0.45 = 0.15$

$P(\mathbf{A}'\cap\mathbf{B}) = P(\mathbf{B}) - P(\mathbf{A}\cap\mathbf{B}) = 0.50 - 0.45 = 0.05$

$P[(\mathbf{A}\cap\mathbf{B}')\cup(\mathbf{A}'\cap\mathbf{B})] = P(\mathbf{A}\cap\mathbf{B}') + P(\mathbf{A}'\cap\mathbf{B}) = 0.15 + 0.05 = 0.20$

$P[(\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cup\mathbf{B})'] = P(\mathbf{A}\cap\mathbf{B}) + P[(\mathbf{A}\cup\mathbf{B})'] = 0.45 + 0.35 = 0.80$

$P[(\mathbf{A}'\cap\mathbf{B})\cup(\mathbf{A}\cup\mathbf{B})'] = P(\mathbf{A}'\cap\mathbf{B}) + P[(\mathbf{A}\cup\mathbf{B})'] = 0.05 + 0.35 = 0.40$

$P[\mathbf{A}\cup(\mathbf{A}\cup\mathbf{B})'] = P(\mathbf{A}) + P[(\mathbf{A}\cup\mathbf{B})'] = 0.60 + 0.35 = 0.95$

$P[\mathbf{B}\cup(\mathbf{A}\cup\mathbf{B})'] = P(\mathbf{B}) + [P(\mathbf{A}\cup\mathbf{B})'] = 0.50 + 0.35 = 0.85$

$P[(\mathbf{A}\cap\mathbf{B}')\cup(\mathbf{A}'\cap\mathbf{B})\cup(\mathbf{A}\cup\mathbf{B})'] = P[(\mathbf{A}\cap\mathbf{B}')] + P[(\mathbf{A}'\cap\mathbf{B})] + P[(\mathbf{A}\cup\mathbf{B})'] = 0.15 + 0.05 + 0.35 = 0.55$

16.
Since $\mathbf{A} \subseteq \mathbf{B}$, $\mathbf{B} = \mathbf{A}\cup(\mathbf{B}\cap\mathbf{A}')$.

Since $\mathbf{A}\cap(\mathbf{B}\cap\mathbf{A}') = \phi$, $P(\mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}\cap\mathbf{A}')$.

Therefore, $P(\mathbf{A}) \le P(\mathbf{A}) + P(\mathbf{B}\cap\mathbf{A}') = P(\mathbf{B})$

17.
$P(\mathbf{A}') = 1 - P(\mathbf{A})$, $P(\mathbf{B}') = 1 - P(\mathbf{B})$

Since $P(\mathbf{A}) \le P(\mathbf{B})$,

$P(\mathbf{B}') = 1 - P(\mathbf{B}) \le 1 - P(\mathbf{A}) = P(\mathbf{A}')$

18.
In Lesson 7, supplementary problem 15a, we showed

$\mathbf{A} \cap \mathbf{C}' \subseteq (\mathbf{A} \cap \mathbf{B}') \cup (\mathbf{B} \cap \mathbf{C}')$

From Supplementary problem 16 above, we conclude,

$P(\mathbf{A} \cap \mathbf{C}') \subseteq P(\mathbf{A} \cap \mathbf{B}') + P(\mathbf{B} \cap \mathbf{C}')$

19.
Step 1: First we will show: $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C} \subseteq \mathbf{A} \cap \mathbf{B}$.

$(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) \cap (\mathbf{A} \cap \mathbf{B})' = (\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) \cap (\mathbf{A}' \cup \mathbf{B}') = (\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) \cap \mathbf{A}' \cup (\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) \cap \mathbf{B}' = \phi \cup \phi = \phi$

Step 2: Since $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C} \subseteq \mathbf{A} \cap \mathbf{B}$, we have $P(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) \leq P(\mathbf{A} \cap \mathbf{B})$

20.
Let **E** be the event that only **A** or **B** occurs but not both.

Step 1: $\mathbf{E} = (\mathbf{A} \cap \mathbf{B}') \cup (\mathbf{A}' \cap \mathbf{B})$

$\mathbf{A} = (\mathbf{A} \cap \mathbf{B}') \cup (\mathbf{A} \cap \mathbf{B})$

$\mathbf{B} = (\mathbf{B} \cap \mathbf{A}') \cup (\mathbf{A} \cap \mathbf{B})$

Step 2: $P(\mathbf{A}) = P(\mathbf{A} \cap \mathbf{B}') + P(\mathbf{A} \cap \mathbf{B})$

$P(\mathbf{A} \cap \mathbf{B}') = P(\mathbf{A}) - P(\mathbf{A} \cap \mathbf{B})$

Step 3: $P(\mathbf{B}) = P(\mathbf{B} \cap \mathbf{A}') + P(\mathbf{A} \cap \mathbf{B})$

$P(\mathbf{A}' \cap \mathbf{B}) = P(\mathbf{B}) - P(\mathbf{A} \cap \mathbf{B})$

Step 4: $P(\mathbf{E}) = P(\mathbf{A} \cap \mathbf{B}') + P(\mathbf{A}' \cap \mathbf{B}) = P(\mathbf{A}) - P(\mathbf{A} \cap \mathbf{B}) + P(\mathbf{B}) - P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - 2P(\mathbf{A} \cap \mathbf{B})$

21.
$P(\mathbf{A}) = 1/3$

$P(\mathbf{B}) = 3/4$.

Step 1: Since $\mathbf{A} \cap \mathbf{B} \subseteq \mathbf{A}$, $P(\mathbf{A} \cap \mathbf{B}) \leq P(\mathbf{A}) = 1/3$.

Therefore, $P(\mathbf{A} \cap \mathbf{B}) \leq 1/3$.

Step 2: $P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \cap \mathbf{B}) = 1/3 + 3/4 - P(\mathbf{A} \cap \mathbf{B})$

Step 3: Since $P(\mathbf{A} \cup \mathbf{B}) \leq 1$,

$1/3 + 3/4 - P(\mathbf{A} \cap \mathbf{B}) \leq 1$

$13/12 - P(\mathbf{A} \cap \mathbf{B}) \leq 1$

$- P(\mathbf{A} \cap \mathbf{B}) \leq 1 - 13/12$

$- P(\mathbf{A} \cap \mathbf{B}) \leq - 1/12$

$P(\mathbf{A} \cap \mathbf{B}) \geq 1/12$

Together, we have

$1/12 \leq P(\mathbf{A} \cap \mathbf{B}) \leq 1/3$

22.

➤a.
False. We can have sets $\mathbf{A} \neq \phi$, where $P(\mathbf{A}) = 0$. For example assume Mr. Smith receives at least 1 phone call a day. Here $\mathbf{S} = \{1,2,3,...\}$. Let $\mathbf{E} = \{1,000,000, 1,000,001, ...\}$. Clearly $P(\mathbf{E}) = 0$, yet $\mathbf{E} \neq \phi$.

➤b.
True. Assume $P(\mathbf{A}) = 0.76$, $P(\mathbf{B}) = 0.92$, and $P(\mathbf{A} \cap \mathbf{B}) = 0.70$. Then

$P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \cap \mathbf{B}) = 0.76 + 0.92 - 0.70$

➤c.
True. See b.

➤d.
False

Step 1: Since $\mathbf{A} \subseteq \mathbf{B}$ then $\mathbf{B} = \mathbf{A} \cup (\mathbf{A}' \cap \mathbf{B})$

Step 2: $P(\mathbf{B}) = P(\mathbf{A}) + P(\mathbf{A}' \cap \mathbf{B})$.

Step 3: Assume $P(\mathbf{A}' \cap \mathbf{B}) = 0$ and $\mathbf{A}' \cap \mathbf{B} \neq \phi$.

Therefore, $\mathbf{A} \subset \mathbf{B}$ and $\mathbf{A} \neq \mathbf{B}$.

➤e.
False. Since $\mathbf{A} \cap \mathbf{B} = \phi$, $P(\mathbf{A}) = 0.76$ and $P(\mathbf{B}) = 0.92$, then

$P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) = 0.76 + 0.92 = 1.68$.

Since $P(\mathbf{A} \cup \mathbf{B}) > 1$, which is not permitted.

➤f.
False. Can $P(\mathbf{A}) = 0.26$ and $P(\mathbf{B}) = 0.52$ and $P(\mathbf{A}\cap\mathbf{B}) = 0.80$?

$P(\mathbf{A}\cup\mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A}\cap\mathbf{B}) = 0.26 + 0.52 - 0.80 = -0.02$

23.
➤a.
We will first show that $\mathbf{E} = (\mathbf{E}\cap\mathbf{D}')\cup(\mathbf{E}\cap\mathbf{D})$.

Step 1: $\mathbf{E}\cap[(\mathbf{E}\cap\mathbf{D}')\cup(\mathbf{E}\cap\mathbf{D})]' = \mathbf{E}\cap[(\mathbf{E}\cap\mathbf{D}')'\cap(\mathbf{E}\cap\mathbf{D})'] = [\mathbf{E}\cap(\mathbf{E}'\cup\mathbf{D})]\cap(\mathbf{E}'\cup\mathbf{D}') =$

$[(\mathbf{E}\cap\mathbf{E}')\cup(\mathbf{E}\cap\mathbf{D})]\cap(\mathbf{E}'\cup\mathbf{D}') = (\mathbf{E}\cap\mathbf{D})\cap(\mathbf{E}'\cup\mathbf{D}') = (\mathbf{E}\cap\mathbf{D})\cap(\mathbf{E}\cap\mathbf{D})' = \phi$

Therefore, $\mathbf{E} \subseteq (\mathbf{E}\cap\mathbf{D}')\cup(\mathbf{E}\cap\mathbf{D})$

Step 2: $\mathbf{E}'\cap[(\mathbf{E}\cap\mathbf{D}')\cup(\mathbf{E}\cap\mathbf{D})] = [\mathbf{E}'\cap(\mathbf{E}\cap\mathbf{D}')]\cup[\mathbf{E}'\cap(\mathbf{E}\cap\mathbf{D})] = \phi \cup \phi = \phi$

Therefore, $(\mathbf{E}\cap\mathbf{D}')\cup(\mathbf{E}\cap\mathbf{D}) \subseteq \mathbf{E}$ and it follows $\mathbf{E} = (\mathbf{E}\cap\mathbf{D}')\cup(\mathbf{E}\cap\mathbf{D})$.

To show $\mathbf{A}\cap\mathbf{B} = (\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C})$, let $\mathbf{E} = \mathbf{A}\cap\mathbf{B}$ and $\mathbf{D} = \mathbf{C}$.

Substitute in above: $\mathbf{E} = (\mathbf{E}\cap\mathbf{D}')\cup(\mathbf{E}\cap\mathbf{D}) = \mathbf{A}\cap\mathbf{B} = (\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C})$

➤b
To show $\mathbf{A} = (\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}')\cup(\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C})$, let $\mathbf{E} = \mathbf{A}$ and $\mathbf{D} = (\mathbf{B}\cup\mathbf{C})$.

Substitute in above: $\mathbf{E} = (\mathbf{E}\cap\mathbf{D}')\cup(\mathbf{E}\cap\mathbf{D}) = \mathbf{A} = [\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C})']\cup[\mathbf{E}\cap(\mathbf{B}\cup\mathbf{C})] =$

$[\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}']\cup[\mathbf{E}\cap(\mathbf{B}\cup\mathbf{C})]$

➤c.
$\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}'$

$\mathbf{A}\cap\mathbf{B} = (\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C})$

$P(\mathbf{A}\cap\mathbf{B}) = P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}') + P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}) = P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}') + 0.125 = 0.25$

$P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}') = 0.125$

Since all these probabilities are the same:

$P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}') = P(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}) = P(\mathbf{A}'\cap\mathbf{B}\cap\mathbf{C}) = 0.125$

$\mathbf{A} = (\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}')\cup[\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C})]$

$P(\mathbf{A}) = P(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}') + P(\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C})) = P(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}') + P[(\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{C})] =$

$P(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}') + P(\mathbf{A}\cap\mathbf{B}) + P(\mathbf{A}\cap\mathbf{C}) - P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}) = 0.50$

$P(\mathbf{A}) = P(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}') + 0.25 + 0.25 - 0.125 = 0.50$

$P(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}') = 0.125$

Since all these probabilities are the same:

$P(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}') = P(\mathbf{A}'\cap\mathbf{B}\cap\mathbf{C}') = P(\mathbf{A}'\cap\mathbf{B}'\cap\mathbf{C}) = 0.125$

$P(\mathbf{A}'\cap\mathbf{B}'\cap\mathbf{C}') = 1 - P(\mathbf{A} \cup \mathbf{B} \cup \mathbf{C})$

From problem 13,

$P(\mathbf{A}\cup\mathbf{B}\cup\mathbf{C}) = P(\mathbf{A}) + P(\mathbf{B}) + P(\mathbf{C}) - P(\mathbf{B}\cap\mathbf{C}) - P(\mathbf{A}\cap\mathbf{B}) - P(\mathbf{A}\cap\mathbf{C}) + P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{A}\cap\mathbf{C})\}$

$0.5 + 0.5 + 0.5 - 0.25 - 0.25 - .25 + 0.125 \quad = 0.875$

$P(\mathbf{A}'\cap\mathbf{B}'\cap\mathbf{C}') = 1 - P(\mathbf{A} \cup \mathbf{B} \cup \mathbf{C}) = 0.875$

24.
$\mathbf{E} = \mathbf{A}_1 \cup\mathbf{A}_2\cup\ldots\cup\mathbf{A}_n$

$\mathbf{E}' = (\mathbf{A}_1 \cup\mathbf{A}_2\cup\ldots\cup\mathbf{A}_n)' = \mathbf{A}_1{}'\cap\mathbf{A}_2{}'\cap\ldots\cap\mathbf{A}_n{}'$

$P(\mathbf{E}) = 1 - P(\mathbf{E}') = 1 - P(\mathbf{A}_1{}'\cap\mathbf{A}_2{}'\cap\ldots\cap\mathbf{A}_n{}')$

25.
Step 1: $\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C}) = (\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{C}) = \phi$.

Therefore, $P[\ \mathbf{A}\cup(\mathbf{B}\cup\mathbf{C})] = P(\mathbf{A}) + P(\mathbf{B}\cup\mathbf{C}) = P(\mathbf{A}) + P(\mathbf{B}) + P(\mathbf{C})$.

26.
$P[\ \mathbf{A}\cup(\mathbf{B}\cup\mathbf{C})] = P(\mathbf{A}) + P(\mathbf{B}\cup\mathbf{C})$

$P(\mathbf{B}\cup\mathbf{C}) = P(\mathbf{B}) + P(\mathbf{C}) - P(\mathbf{B}\cap\mathbf{C})$

Therefore, $P(\mathbf{A}\cup\mathbf{B}\cup\mathbf{C}) = P[\ \mathbf{A}\cup(\mathbf{B}\cup\mathbf{C})] = P(\mathbf{A}) + P(\mathbf{B}\cup\mathbf{C}) = \quad P(\mathbf{A}) + P(\mathbf{B}) + P(\mathbf{C}) - P(\mathbf{B}\cap\mathbf{C})$

27.
Step 1: Since each of four sets have no common intersection,

$P(\mathbf{B}\cap\mathbf{C}) + P(\mathbf{B}'\cap\mathbf{C}) + P(\mathbf{B}\cap\mathbf{C}') + P(\mathbf{B}'\cap\mathbf{C}') = P[(\mathbf{B}\cap\mathbf{C})\cup(\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{B}'\cap\mathbf{C}')]$

From the associative law:

$[(\mathbf{B}\cap\mathbf{C})\cup(\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{B}'\cap\mathbf{C}') = [(\mathbf{B}\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{C}')]\cup[(\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{B}'\cap\mathbf{C}')]$

From the distributive law:

$[(\mathbf{B}\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{C}')]\cup[(\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{B}'\cap\mathbf{C}')] = [(\mathbf{B}\cap(\mathbf{C}\cup\mathbf{C}')]\cup[(\mathbf{B}'\cap(\mathbf{C}\cup\mathbf{C}')] = (\mathbf{B}\cap\mathbf{S})\cup(\mathbf{B}'\cap\mathbf{S}) =$

$\mathbf{B}\cup\mathbf{B}' = \mathbf{S}$

Therefore,

$P(\mathbf{B}\cap\mathbf{C}) + P(\mathbf{B}'\cap\mathbf{C}) + P(\mathbf{B}\cap\mathbf{C}') + P(\mathbf{B}'\cap\mathbf{C}') = P[(\mathbf{B}\cap\mathbf{C})\cup(\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{B}'\cap\mathbf{C}')] = P(\mathbf{S}) = 1.$

28.
$P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}) + P(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}) + P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}') + P(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}') =$

$P[(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}')] =$

$P[\mathbf{A}\cap[(\mathbf{B}\cap\mathbf{C})\cup(\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{B}'\cap\mathbf{C}')] = P(\mathbf{A}\cap\mathbf{S}) = P(\mathbf{A})$

29.
Since $\mathbf{A}'\cap\mathbf{B}' = \phi$, $(\mathbf{A}'\cap\mathbf{B}')' = \phi' = \mathbf{A}\cup\mathbf{B} = \mathbf{S}$

$P(\mathbf{A}\cup\mathbf{B}) = P(\mathbf{S}) = 1$

$P(\mathbf{A}\cup\mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A}\cap\mathbf{B}) = 1$

$P(\mathbf{A}) + P(\mathbf{B}) = 1 + P(\mathbf{A}\cap\mathbf{B})$

30.
$\mathbf{S} = \{1,2,3,4,5,6,7,8,9,10\}$

$\mathbf{A} = \{4,5,6,7,8,9,10\}$, $\mathbf{B} = \{1,2,3,4,7,8,9,10\}$

$\mathbf{A}' = \{1,2,3\}$, $\mathbf{B} = \{5,6\}$