Set Theory
Lesson 9
Boolean Algebra of Sets

9.1- What is the Boolean Algebra of sets?

9.1 - Problem 1:

➤(a).

We can write the expression *all the chips are defective* as

1. $\mathbf{D_1}$ = The first chip is defective

and

2. $\mathbf{D_2}$ = The second chip is defective

and

3. $\mathbf{D_3}$ = The third chip is defective

and

4. $\mathbf{D_4}$ = The fourth chip is defective

We connect the events in Step 2 using ∩ in place of *and*:

5. $\mathbf{E} = \mathbf{D_1 \cap D_2 \cap D_3 \cap D_4}$.

➤(b).

The expression *only one chip is defective* means that exactly one is defective and the other three chips are not defective. This can happen in the following ways:

1. The first chip is defective and the other three are not:

$\mathbf{D_1 \cap D_2' \cap D_3' \cap D_4'}$

or

2. The second chip is defective and the other three are not:

$\mathbf{D_1' \cap D_2 \cap D_3' \cap D_4'}$

or

3. The third chip is defective and the other three are not:

$\mathbf{D_1' \cap D_2' \cap D_3 \cap D_4'}$

or

4. The fourth chip is defective and the other three are not:

$\mathbf{D_1' \cap D_2' \cap D_3' \cap D_4}$

Since these four events are connected by *or*, we write the expression *only one chip is defective* as

5. $\mathbf{E = (D_1 \cap D_2' \cap D_3' \cap D_4') \cup (D_1' \cap D_2 \cap D_3' \cap D_4') \cup (D_1' \cap D_2' D_3 \cap D_4') \cup (D_1' \cap D_2' \cap D_3' \cap D_4)}$

➤(c).
We can write the expression *none of the chips are defective* as

1. $\mathbf{D_1'}$ = The first chip is not defective

and

2. $\mathbf{D_2'}$ = The second chip is not defective

and

3. $\mathbf{D_3'}$ = The third chip is not defective

and

4. $\mathbf{D_4'}$ = The fourth chip is not defective

We connect the events in Step 2 using ∩ in place of *and*:

5. $\mathbf{E = D_1' \cap D_2' \cap D_3' \cap D_4'}$.

➤(d).
Step 1: The compliment of the event *at least one chip is defective* is none of the chips are defective.

Step 2: From (c), we have the event none of the chips are defective written as

$\mathbf{D_1' \cap D_2' \cap D_3' \cap D_4'}$.

Step 3: Now the compliment of the event *none of the chips are defective* is

at least one chip is defective. From step 2, we have

$\mathbf{E} = (\mathbf{D_1}'\cap\mathbf{D_2}'\cap\mathbf{D_3}'\cap\mathbf{D_4}')'$.

Step 4: From DeMorgan's Law we have

$\mathbf{E} = (\mathbf{D_1}'\cap\mathbf{D_2}'\cap\mathbf{D_3}'\cap\mathbf{D_4}')' = \mathbf{D_1}''\cup\mathbf{D_2}''\cup\mathbf{D_3}''\cup\mathbf{D_4}'' = \mathbf{D_1}\cup\mathbf{D_2}\cup\mathbf{D_3}\cup\mathbf{D_4}$.

➤(e).
Using the Distributive law

$(\mathbf{D_1}\cap\mathbf{D_2}\cap\mathbf{D_3}\cap\mathbf{D_4})\cap[\mathbf{D_1}'\cup\mathbf{D_2}'\cup\mathbf{D_3}'\cup\mathbf{D_4}'] =$

$[(\mathbf{D_1}\cap\mathbf{D_2}\cap\mathbf{D_3}\cap\mathbf{D_4})\cap\mathbf{D_1}']\cup[(\mathbf{D_1}\cap\mathbf{D_2}\cap\mathbf{D_3}\cap\mathbf{D_4})\cap\mathbf{D_2}']\cup[(\mathbf{D_1}\cap\mathbf{D_2}\cap\mathbf{D_3}\cap\mathbf{D_4})\cap\mathbf{D_3}']\cup[(\mathbf{D_1}\cap\mathbf{D_2}\cap\mathbf{D_3}\cap\mathbf{D_4})\cap\mathbf{D_4}']$

$= \phi\cup\phi\cup\phi\cup\phi = \phi$

➤(f).
Using the Distributive law

$(\mathbf{D_1}\cup\mathbf{D_2}\cup\mathbf{D_3}\cup\mathbf{D_4})\cup[\mathbf{D_1}'\cap\mathbf{D_2}'\cap\mathbf{D_3}'\cap\mathbf{D_4}'] =$

$[(\mathbf{D_1}\cup\mathbf{D_2}\cup\mathbf{D_3}\cup\mathbf{D_4})\cup\mathbf{D_1}']\cap[(\mathbf{D_1}\cup\mathbf{D_2}\cup\mathbf{D_3}\cup\mathbf{D_4})\cup\mathbf{D_2}']\cap[(\mathbf{D_1}\cup\mathbf{D_2}\cup\mathbf{D_3}\cup\mathbf{D_4})\cup\mathbf{D_3}']\cap[(\mathbf{D_1}\cup\mathbf{D_2}\cup\mathbf{D_3}\cup\mathbf{D_4})\cup\mathbf{D_4}']$

$= \mathbf{S}\cap\mathbf{S}\cap\mathbf{S}\cap\mathbf{S} = \mathbf{S}$

9.1 - Problem 2:
➤(a).
This can happen in three different ways:

1. The first and second tube came from the east and the third from the west coast:

$(\mathbf{E_1}\cap\mathbf{E_2}\cap\mathbf{W_3})$

or

2. The first and third tube came from the east and the second from the west coast:

$(\mathbf{E_1}\cap\mathbf{W_2}\cap\mathbf{E_3})$

or

3. The second and third tube came from the east and the first from the west coast:

$(\mathbf{W_1} \cap \mathbf{E_2} \cap \mathbf{E_3})$

Since each of these is connected by *or* we have

4. $\mathbf{F} = (\mathbf{E}_1 \cap \mathbf{E}_2 \cap \mathbf{W_3}) \cup (\mathbf{E_1} \cap \mathbf{W_2} \cap \mathbf{E_3}) \cup (\mathbf{W_1} \cap \mathbf{E_2} \cap \mathbf{E_3})$.

➤(b).
This means that two of the tubes came from the Midwest and the third did not come from the Midwest:

1. The first and second came from the Midwest and the third did not:

$(\mathbf{M}_1 \cap \mathbf{M}_2 \cap \mathbf{M_3}')$

or

2. The first and third came from the Midwest and the second did not:

$(\mathbf{M}_1 \cap \mathbf{M}_2' \cap \mathbf{M_3})$

or

3. The second and third came from the Midwest and the first did not:

$(\mathbf{M}_1' \cap \mathbf{M}_2 \cap \mathbf{M_3})$

4. Since these are connected by *or* we have

$\mathbf{F} = (\mathbf{M}_1 \cap \mathbf{M}_2 \cap \mathbf{M_3}') \cup (\mathbf{M}_1 \cap \mathbf{M}_2' \cap \mathbf{M_3}) \cup (\mathbf{M}_1' \cap \mathbf{M}_2 \cap \mathbf{M_3})$

➤(c).
1. $\mathbf{W_1}'$ = First tube did not come the West coast.

and

2. $\mathbf{W_2}'$ = Second tube did not come the West coast.

and

3. $\mathbf{W_3}'$ = Third tube did not come the West coast.

4. Since all three events are connected by *and* we have **F=** $W_1' \cap W_2' \cap W_3'$

➤(d).
1. W_1 = First tube came from the West coast.

and

2. W_2 = Second tube came from the West coast.

and

3. E_3 = Third tube came from the East coast.

4. Since all three events are connected by *and* we have

F= $W_1 \cap W_2 \cap E_3$

➤(e).
This event can happen in six different ways:

1. First tube came from West coast **and** second tube came from East coast **and** third tube came from the Midwest = $W_1 \cap E_2 \cap M_3$.

or

2. First tube came from West coast **and** second tube came from Midwest **and** third tube came from the East coast = $W_1 \cap M_2 \cap E_3$.

or

3. First tube came from East coast **and** second tube came from West coast **and** third tube came from the Midwest = $E_1 \cap W_2 \cap M_3$.

or

4. First tube came from East coast **and** second tube came from Midwest **and** third tube came from the West coast = $E_1 \cap M_2 \cap W_3$.

or

5. First tube came from Midwest **and** second tube came from West coast **and** third tube came from the East coast = $M_1 \cap W_2 \cap E_3$.

or

6. First tube came from Midwest **and** second tube came from East coast **and** third tube came

from the West coast = $\mathbf{M_1 \cap E_2 \cap W_3}$.

7. Since each of these expressions are connected by *or* we have

$$\mathbf{F} = (\mathbf{W_1 \cap E_2 \cap M_3}) \cup (\mathbf{W_1 \cap M_2 \cap E_3}) \cup (\mathbf{E_1 \cap W_2 \cap M_3}) \cup (\mathbf{E_1 \cap M_2 \cap W_3}) \cup (\mathbf{M_1 \cap W_2 \cap E_3}) \cup (\mathbf{M_1 \cap E_2 \cap W_3})$$

9.1 - Problem 1.3:

➤(a).

Step 1: We can write the event that the car is traveling at least 75 mph as {75, 76, 77, ...}

Step 2: **A'** = {45,46,47,...,74}' = {75,76,77,...} = **E**

➤(b).

Step 1: **B** = {47,48,49,...}

Step 2: **B' =** {47,48,49,...}**' =** {45,46}.

➤(c).

Step 1: **A** = {45,46,47,...,74}

Step 2: **B** = {47,48,49,...}

Step 3: **A**∩**B** = {45,46,47,...,74}∩{47,48,49,...} = {47,48,...,74}.

➤(d).

Step 1: **A** = {45,46,47,...,74}

Step 2: **C** = {74,75,76,...,80}

Step 3: **A**∩**B** = {45,46,47,...,74}∩{74,75,76,...,80} = {74}.

9.1 - Problem 4:

The event *a tube is tested and found flawed,* can be stated as follows:

1. A tube is tested and found flawed *and* it came from supplier **A**

T∩A

or

2. A tube is tested and found flawed *and* it came from supplier **B**

T∩B

or

3. A tube is tested and found flawed *and* it came from supplier **C.**

$T \cap C$

Since these are connected with *or* we have

$T = (T \cap A) \cup (T \cap B) \cup (T \cap C)$

9.1 - Problem 5:
The event that a six occurs on the second toss can happen in the following ways:

1. A six occurs on the second toss *and a six occurs on the first toss.*

$S_1 \cap S_2$

or

2. A six occurs on the second toss *and a six does not occurs on the first toss.*

$S_1' \cap S_2$

Since these events are connected with an *or we have*

$S_2 = (S_1 \cap S_2) \cup (S_1' \cap S_2)$

Supplementary Problems

1.
The first DeMorgan law is

$(A \cup B)' = A' \cap B'$
We extend this equation to three sets:

$(A \cup B \cup C)' = A' \cap B' \cap C'$
The second DeMorgan law is

$(A \cap B)' = A' \cup B'$

We extend this equation to three sets:

$(A \cap B \cap C)' = A' \cup B' \cup C'$

2.
$\mathbf{E} = [(\mathbf{D}_1\cup\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)]\cap[\mathbf{D}_1\cup(\mathbf{D}_2'\cap\mathbf{D}_3'\cap\mathbf{D}_4')]$

Step 1: We use the distributive law in reverse order

$(\mathbf{A}\cup\mathbf{B})\cap(\mathbf{A}\cup\mathbf{C}) = \mathbf{A}\cup(\mathbf{B}\cap\mathbf{C})$

Step 2:

$\mathbf{E} = [(\mathbf{D}_1\cup\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)]\cap[\mathbf{D}_1\cup(\mathbf{D}_2'\cap\mathbf{D}_3'\cap\mathbf{D}_4')] = [\mathbf{D}_1\cup(\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)]\cap[\mathbf{D}_1\cup(\mathbf{D}_2'\cap\mathbf{D}_3'\cap\mathbf{D}_4')]$

Step 3: Let $\mathbf{A} = \mathbf{D}_1$ for Step 2:

$\mathbf{E} = [(\mathbf{D}_1\cup\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)]\cap[\mathbf{D}_1\cup(\mathbf{D}_2'\cap\mathbf{D}_3'\cap\mathbf{D}_4')] = [\mathbf{D}_1\cup(\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)]\cap\{\mathbf{D}_1\cup(\mathbf{D}_2'\cap\mathbf{D}_3'\cap\mathbf{D}_4')\} =$

$= \mathbf{D}_1\cup[(\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)\cap(\mathbf{D}_2'\cap\mathbf{D}_3'\cap\mathbf{D}_4')]$

Step 4: Using DeMorgan's law $(\mathbf{A}'\cap\mathbf{B}') = (\mathbf{A}\cup\mathbf{B})'$:

$\mathbf{E} = \mathbf{D}_1\cup[(\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)\cap(\mathbf{D}_2'\cap\mathbf{D}_3'\cap\mathbf{D}_4')] = \mathbf{D}_1\cup[(\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)\cap(\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)'$

Step 5: Using the Complimentary law: $\mathbf{A}\cap\mathbf{A}' = \phi$ and identity law $\mathbf{A}\cup\phi = \mathbf{A}$:

$\mathbf{E} = \mathbf{D}_1\cup[(\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)\cap(\mathbf{D}_2\cup\mathbf{D}_3\cup\mathbf{D}_4)' = \mathbf{D}_1\cup\phi = \mathbf{D}_1$.

$\mathbf{E} = \mathbf{D}_1$

3.
➤(a).
Since **E** is the event that *three tosses occurred*, two 6s' must occur where the second 6 occurred on the third toss:

1. A six occurred on the first and third toss but not on the second toss:

$\mathbf{A}_1\cap\mathbf{A}_2'\cap\mathbf{A}_3$

or

2. A six occurred on the second and third toss but no on the first toss:

$\mathbf{A}_1'\cap\mathbf{A}_2\cap\mathbf{A}_3$

3. Since these events are connected by an *or* we have

$\mathbf{E} = (\mathbf{A_1} \cap \mathbf{A_2}' \cap \mathbf{A_3}) \cup (\mathbf{A_1}' \cap \mathbf{A_2} \cap \mathbf{A_3})$

➤(b).

The event *there was at most three tosses,* means there occurred two or three tosses:

1. A six occurred on the first and second toss:

$(\mathbf{A_1} \cap \mathbf{A_2})$

or

2. A six did not occur on the first toss but a six occurred on the second and third toss

$\mathbf{A_1}' \cap \mathbf{A_2} \cap \mathbf{A_3}$

or

3. A six occurred on the first toss and the third toss but did not occur on the second toss:

$(\mathbf{A_1} \cap \mathbf{A_2}' \cap \mathbf{A_3})$

4. Since each of the above events are connect by *or*, we have

$\mathbf{E} = (\mathbf{A_1} \cap \mathbf{A_2}) \cup (\mathbf{A_1}'\mathbf{A_2} \cap \mathbf{A_3}) \cup (\mathbf{A_1} \cap \mathbf{A_2}' \cap \mathbf{A_3})$

➤(c).

Step 1: Let **E** = *there was four tosses.*

Step 2: **E**' = the event *there was at most three tosses.*

Step 3: From (b), $\mathbf{E}' = (\mathbf{A_1} \cap \mathbf{A_2}) \cup (\mathbf{A_1}'\mathbf{A_2} \cap \mathbf{A_3}) \cup (\mathbf{A_1} \cap \mathbf{A_2}' \cap \mathbf{A_3})$

Step 4: $\mathbf{E}'' = \mathbf{E} = [(\mathbf{A_1} \cap \mathbf{A_2}) \cup (\mathbf{A_1}'\mathbf{A_2} \cap \mathbf{A_3}) \cup (\mathbf{A_1} \cap \mathbf{A_2}' \cap \mathbf{A_3})]'$

4.

1. DeMorgan's Law: $(\mathbf{E} \cup \mathbf{F})' = \mathbf{E}' \cap \mathbf{F}'$:

$\{(\mathbf{A}' \cap \mathbf{B}') \cup [\mathbf{C} \cup (\mathbf{D} \cap \mathbf{E})]' \cup [\mathbf{F}' \cap (\mathbf{G}' \cup \mathbf{H}')]\}' = \{(\mathbf{A}' \cap \mathbf{B}')' \cap [\mathbf{C} \cup (\mathbf{D} \cap \mathbf{E})]'' \cap [\mathbf{F}' \cap (\mathbf{G}' \cup \mathbf{H}')]'\}$

2. DeMorgan's Law: $(\mathbf{E} \cap \mathbf{F})' = \mathbf{E}' \cup \mathbf{F}'$:

$\{(\mathbf{A}' \cap \mathbf{B}')' \cap [\mathbf{C} \cup (\mathbf{D} \cap \mathbf{E})]'' \cap [\mathbf{F}' \cap (\mathbf{G}' \cup \mathbf{H}')]'\} = \{(\mathbf{A}'' \cup \mathbf{B}'') \cap [\mathbf{C} \cup (\mathbf{D} \cap \mathbf{E})] \cup [\mathbf{F}'' \cup (\mathbf{G}' \cup \mathbf{H}')']\}$

3. DeMorgan's Law: $(\mathbf{E} \cup \mathbf{F})' = \mathbf{E}' \cap \mathbf{F}'$:

$\{(\mathbf{A}''\cup\mathbf{B}'')\cap[\mathbf{C}\cup(\mathbf{D}\cap\mathbf{E})]\cup[\mathbf{F}''\cup(\mathbf{G}'\cup\mathbf{H}')']\} = \{(\mathbf{A}\cup\mathbf{B})\cap[\mathbf{C}\cup(\mathbf{D}\cap\mathbf{E})]\cup[\mathbf{F}\cup(\mathbf{G}''\cap\mathbf{H}'')]\} =$

$\{(\mathbf{A}\cup\mathbf{B})\cap[\mathbf{C}\cup(\mathbf{D}\cap\mathbf{E})]\cup[\mathbf{F}\cup(\mathbf{G}\cap\mathbf{H})]\}$

5.
We expand the Distributive law $\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C}) = (\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{B})$ to

$\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C}\cup\mathbf{D}\cup\mathbf{E}) = (\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{D})\cup(\mathbf{A}\cap\mathbf{E})$

6.
We expand the Distributive law $\mathbf{A}\cup(\mathbf{B}\cap\mathbf{C}) = (\mathbf{A}\cup\mathbf{B})\cap(\mathbf{A}\cup\mathbf{B})$ to

$\mathbf{A}\cup(\mathbf{B}\cap\mathbf{C}\cap\mathbf{D}\cap\mathbf{E}) = (\mathbf{A}\cup\mathbf{B})\cap(\mathbf{A}\cup\mathbf{C})\cap(\mathbf{A}\cup\mathbf{D})\cap(\mathbf{A}\cup\mathbf{E})$

For questions 7 & 8, prove the identity:

7.
Step 1: Use the Distributive law $\mathbf{E}\cap(\mathbf{F}\cup\mathbf{G}) = (\mathbf{F}\cup\mathbf{G})\cap\mathbf{E} = (\mathbf{E}\cap\mathbf{F})\cup(\mathbf{E}\cap\mathbf{G})$:

$(\mathbf{A}\cup\mathbf{B})\cap(\mathbf{C}\cup\mathbf{D}) = [(\mathbf{A}\cup\mathbf{B})\cap\mathbf{C}]\cup[(\mathbf{A}\cup\mathbf{B})\cap\mathbf{D}] = [(\mathbf{A}\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{C})]\cup[(\mathbf{A}\cap\mathbf{D})\cup(\mathbf{B}\cap\mathbf{D})]$

Step 2: Use the Commutative law $\mathbf{E}\cup\mathbf{F} = \mathbf{F}\cup\mathbf{E}$:

$(\mathbf{A}\cup\mathbf{B})\cap(\mathbf{C}\cup\mathbf{D}) = [(\mathbf{A}\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{C})]\cup[(\mathbf{A}\cap\mathbf{D})\cup(\mathbf{B}\cap\mathbf{D})] = (\mathbf{A}\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{D})\cup(\mathbf{B}\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{D})$

8.
Step 1: Use the Distributive law $\mathbf{E}\cup(\mathbf{F}\cap\mathbf{G}) = (\mathbf{F}\cap\mathbf{G})\cup\mathbf{E} = (\mathbf{E}\cup\mathbf{F})\cap(\mathbf{E}\cup\mathbf{G})$:

$(\mathbf{A}\cap\mathbf{B})\cup(\mathbf{C}\cap\mathbf{D}) = [(\mathbf{A}\cap\mathbf{B})\cup\mathbf{C}]\cap[(\mathbf{A}\cap\mathbf{B})\cup\mathbf{D}] = [(\mathbf{A}\cup\mathbf{C})\cap(\mathbf{B}\cup\mathbf{C})]\cap[(\mathbf{A}\cup\mathbf{D})\cap(\mathbf{B}\cup\mathbf{D})]$

Step 2: Use the Commutative law $\mathbf{E}\cap\mathbf{F} = \mathbf{F}\cap\mathbf{E}$:

$(\mathbf{A}\cap\mathbf{B})\cup(\mathbf{C}\cap\mathbf{D}) = [(\mathbf{A}\cup\mathbf{C})\cap(\mathbf{B}\cup\mathbf{C})]\cap[(\mathbf{A}\cup\mathbf{D})\cap(\mathbf{B}\cup\mathbf{D})] = (\mathbf{A}\cup\mathbf{C})\cap(\mathbf{A}\cup\mathbf{D})\cap(\mathbf{B}\cup\mathbf{C})\cap(\mathbf{B}\cup\mathbf{D})$

9.
Step 1: Since $\mathbf{A}\subset\mathbf{B}\subset\mathbf{C}\subset\mathbf{D}\subset\mathbf{E}$ it follows that $\mathbf{E}'\subset\mathbf{D}'\subset\mathbf{C}'\subset\mathbf{B}'\subset\mathbf{A}'$

Step 2: $(\mathbf{B}\cap\mathbf{A}')\cup(\mathbf{C}\cap\mathbf{B}')\cup(\mathbf{D}\cap\mathbf{C}')\cup(\mathbf{E}\cap\mathbf{D}') = [(\mathbf{B}\cap\mathbf{A}')\cup(\mathbf{C}\cap\mathbf{B}')]\cup[(\mathbf{D}\cap\mathbf{C}')\cup(\mathbf{E}\cap\mathbf{D}')]$

Step 3: From problems 7 & 8

$[(\mathbf{B}\cap\mathbf{A}')\cup(\mathbf{C}\cap\mathbf{B}')] = (\mathbf{B}\cup\mathbf{C})\cap(\mathbf{B}\cup\mathbf{B}')\cap(\mathbf{A}'\cup\mathbf{C})\cap(\mathbf{A}'\cup\mathbf{B}') = \mathbf{C}\cap\mathbf{S}\cap(\mathbf{A}'\cup\mathbf{C})\cap\mathbf{A}' =$

$\mathbf{C}\cap[(\mathbf{A'}\cap\mathbf{A'})\cup(\mathbf{A'}\cap\mathbf{C})] = \mathbf{C}\cap[\mathbf{A'}\cup(\mathbf{A'}\cap\mathbf{C})] = \mathbf{C}\cap[(\mathbf{A'}\cup\mathbf{A'})\cap(\mathbf{A'}\cap\mathbf{C})] = \mathbf{C}\cap[\mathbf{A'}\cap(\mathbf{A'}\cap\mathbf{C})] = \mathbf{A'}\cap\mathbf{C}$

Step 4: From problems 7 & 8

$[(\mathbf{D}\cap\mathbf{C'})\cup(\mathbf{E}\cap\mathbf{D'})] = (\mathbf{D}\cup\mathbf{E})\cap(\mathbf{D}\cup\mathbf{D'})\cap(\mathbf{C'}\cup\mathbf{E})\cap(\mathbf{C'}\cup\mathbf{D'}) = \mathbf{E}\cap\mathbf{S}\cap(\mathbf{C'}\cup\mathbf{E})\cap\mathbf{C'} =$

$\mathbf{E}\cap[(\mathbf{C'}\cap\mathbf{C'})\cup(\mathbf{C'}\cap\mathbf{E})] = \mathbf{E}\cap[\mathbf{C'}\cup(\mathbf{C'}\cap\mathbf{E})] = \mathbf{E}\cap[(\mathbf{C'}\cup\mathbf{C'})\cap(\mathbf{C'}\cap\mathbf{E})] = \mathbf{E}\cap[\mathbf{C'}\cap(\mathbf{C'}\cap\mathbf{E})] = \mathbf{C'}\cap\mathbf{E}$

Step 5: $[(\mathbf{B}\cap\mathbf{A'})\cup(\mathbf{C}\cap\mathbf{B'})]\cup[(\mathbf{D}\cap\mathbf{C'})\cup(\mathbf{E}\cap\mathbf{D'})] = (\mathbf{A'}\cap\mathbf{C})\cup(\mathbf{C'}\cap\mathbf{E}) = \mathbf{E}\cap\mathbf{A'}$.

10.

➤ a.

The event **E** *exactly two of the marbles drawn are blue* means that two of the marbles are blue and one is red. $\mathbf{R}_i'$ represents the ith marble is blue.

1. The first two are blue and the third is red: $\mathbf{R}_1'\cap\mathbf{R}_2'\cap\mathbf{R}_3$.

or

2. The first blue and third are blue and the second is red: $\mathbf{R}_1'\cap\mathbf{R}_2\cap\mathbf{R}_3'$.

or

3. The first red is and the second and third is blue: $\mathbf{R}_1\cap\mathbf{R}_2'\cap\mathbf{R}_3$.

Since these events are connected by *or* we have

$\mathbf{E} = (\mathbf{R}_1'\cap\mathbf{R}_2'\cap\mathbf{R}_3)\cup(\mathbf{R}_1'\cap\mathbf{R}_2\cap\mathbf{R}_3')\cup(\mathbf{R}_1\cap\mathbf{R}_2'\cap\mathbf{R}_3)$

The event *exactly two of the marbles drawn are blue*

➤ b.

Step 1: Let **E** = *at most two of the marbles drawn are red*.

Step 2: **E'** = the event that exactly all three marbles drawn are red.

Step 3: $\mathbf{E'} = \mathbf{R}_1\cap\mathbf{R}_2\cap\mathbf{R}_3$

Step 4: $\mathbf{E} = \mathbf{E''} = (\mathbf{R}_1\cap\mathbf{R}_2\cap\mathbf{R}_3)' = \mathbf{R}_1'\cup\mathbf{R}_2'\cup\mathbf{R}_3'$.

➤ c.

The event **E** that *exactly two of the marbles are the same color* means that exactly two red or two blue marbles were drawn.

Step 1: **E'** is the event that all three marbles are the same color:

1. All three marbles selected are red: $\mathbf{R_1 \cap R_2 \cap R_3}$
or

2. All three marbles selected are blue: $\mathbf{R_1' \cap R_2' \cap R_3'}$

Therefore, $\mathbf{E'} = (\mathbf{R_1 \cap R_2 \cap R_3}) \cup (\mathbf{R_1' \cap R_2' \cap R_3'})$.

Step 2: $\mathbf{E} = \mathbf{E''} = [(\mathbf{R_1 \cap R_2 \cap R_3}) \cup (\mathbf{R_1' \cap R_2' \cap R_3'})]' = (\mathbf{R_1' \cup R_2' \cup R_3'}) \cap (\mathbf{R_1 \cup R_2 \cup R_3})$

➤ d.
The event **E** that *a marble drawn on the second drawing is red* says nothing about the first and third draw. Therefore, $\mathbf{E} = \mathbf{R_2}$.

11.
➤ a.
E = *the event that a person will have a hamburger*
or
the event that a person will have a frankfurter.
= $\mathbf{C \cup D}$.

➤ b.
E = the event *a person will order neither a hamburger nor a frankfurter at this restaurant* equals *a person will not order a hamburger and will not order a frankfurter at this restaurant.*

1. A person will not order a hamburger: $\mathbf{C'}$.

and

2. A person will not order a frankfurter: $\mathbf{D'}$.

Therefore, $\mathbf{E} = \mathbf{C' \cap D'}$

➤ c.
$\mathbf{C'}$
➤ d.
E = *a person will order a hamburger but not a frankfurter* equals

a person will order a hamburger and will not order a frankfurter.

= $C \cap \mathbf{D'}$

➤ e.
E = *a person will order a frankfurter but not a hamburger* equals

a person will order a frankfurter and will not order a hamburger.

= $C' \cap D$.

➤ f.
E = *a person will order one or the other but not both* :

1. A person will order a hamburger and not a frankfurter

$C \cap D'$

or

2. A person will order a frankfurter and not a hamburger

$C' \cap D$

$E = (C \cap D') \cup (C' \cap D)$

12.
➤ a.
1. The driver receives one traffic citation: T_1

or

2. The driver receives two traffic citations: T_2

$E = T_1 \cap T_2$.

➤ b.
Step 1: **E** = *receive at most one traffic citation*

Step 2: **E'** = *receives two or more traffic citations:*

1. Receives two citations: T_2

or

2. Receives three citations: T_3

or

3. Receives four citations: T_4

4. Receives five or more: T_5

5. $\mathbf{E'} = \mathbf{T_2 \cup T_3 \cup T_4 \cup T_5}$

6. $\mathbf{E = E''} = (\mathbf{T_2 \cup T_3 \cup T_4 \cup T_5})' = \mathbf{T_2' \cap T_3' \cap T_4' \cap T_5'}$

➤ c.
Let **E** = *receive at least three traffic citations* = receives three or four or five or more traffic citations.

1. receives three citations:$\mathbf{T_3}$

or

2. receives four citations:$\mathbf{T_4}$

or

3. receives five or more citations:$\mathbf{T_5}$

Therefore, $\mathbf{E} = \mathbf{T_3 \cup T_4 \cup T_5}$

➤ d.
Step 1: **E** = *receives no traffic citations*

Step 2: **E'** = *receives at least one citation.*

Step 3: $\mathbf{E'} = \mathbf{T_1 \cup T_2 \cup T_3 \cup T_4 \cup T_5}$

Step 4: $\mathbf{E} = \mathbf{E''} = (\mathbf{T_1 \cup T_2 \cup T_3 \cup T_4 \cup T_5})' = \mathbf{T_1' \cap T_2' \cap T_3' \cap T_4' \cap T_5}$

➤ e.
The event **E** *will not get three traffic citations* says nothing about the number of citations other then the driver will not get three traffic citations. Therefore, $\mathbf{E} = \mathbf{T_3'}$**.**

13.
➤ a.
The sample space **S** = {0,1,2,3,4,...} and **Ck** = {k,k+1,k+2,...}.

Since k = 1, then **C1** means that she makes at least 1 phone call:

C1 = {1,2,3,4,5,...}.

➤ b.
Step 1: **C5** = {5,6,7,8,...}.

Step 2: **C5'** = {5,6,7,8,...}' = {0,1,2,3,4}

Therefore, **C5'** means that she made at most 4 call.

➤ c.
Step 1: **C7 =** {7,8,9,10,...}

Step 2: **C8'** = {0,1,2,3,4,5,6,7}.

Step 3: **C7** ∩**C8**' = {7,8,9,10,...}∩{0,1,2,3,4,5,6,7} **=** {**7**}.

Therefore, **C7** ∩**C8**' means that she made exactly 7 calls.

➤ d.
Step 1: **C10** = {10,11,12,13,...}.

Step 2: **C1'** = {1,2,3,4,5,...}' = {0}.

Step 3: **C10**∪**C1'** = {10,11,12,13,...}∪{0}

Step 4: Therefore, **C10**∪**C1**' means that she made no calls or at least 10 calls.

➤ e.
Step 1: **C10'** = {10,11,12,13,...}' = {0,1,2,3,4,5,6,7,8,9}

Step 2: **C9 = {**9,10,11,...**}**

Step 3: **C10**'∩**C9 =** {0,1,2,3,4,5,6,7,8,9}∩**{**9,10,11,...} **=** {**9**}

Therefore, **C10'**∩**C9** means that she made exactly 9 calls.

➤f.
Step 1: **C10'**= {10,11,12,13,...} = {0,1,2,3,4,5,6,7,8,9}.

Step 2: **C12 =** {12,13,14,**...}**

Step 3: **C10**'∪**C12 =** {0,1,2,3,4,5,6,7,8,9**}**∪**{**12,13,14,...}

Therefore, **C10**'∪**C**12 means that she made at most 9 calls or at least 12 calls.

➤ g.
C1' = {1,2,3,4,5,..}' = {0}.

Therefore, **C1'** means that she made no calls.

➤ h.
Step 1: (**C8'**∪**C15**)' = **C8''**∩**C15' = C8**∩**C15'**

Step 2: **C**8 = {8,9,10,...}

Step 3: **C**15' = {15,16,17,...}' = {0,1,2,3,4,...14}.

Step 4: **C8**∩**C15' = {**8,9,10,...}∩{0,1,2,3,4,...14} = {8,9,10,...14}

Therefore,(**C8'**∪**C15**)' means that she made between 8 and 14 calls a day.

14.
➤ a.
Let **E** **=** *all cards drawn are kings.*

1. First card is a king: $\mathbf{K_1}$

and

2. Second card is a king: $\mathbf{K_2}$

and

3. Third card is a king: $\mathbf{K_3}$

Therefore, **E** = $\mathbf{K_1 \cap K_2 \cap k_3}$

➤ b.
Let **E** **=** *no cards drawn are kings.*

1. First card is not a king: $\mathbf{K_1}'$

and

2. Second card is not a king: $\mathbf{K_2}'$

and

3. Third card is not a king: $\mathbf{K_3}'$

Therefore, $\mathbf{E} = \mathbf{K_1'} \cap \mathbf{K_2'} \cap \mathbf{k_3'}$

➤ c.
Let **E** = *at least two cards drawn are queens.*

Step 1: **E** = exactly two cards drawn are queens or all three are queens.

Step 2: Let $\mathbf{E_1}$ = exactly two cards drawn are queens.

Step 3: Let $\mathbf{E_2}$ = exactly three cards drawn are queens.

Step 4: $\mathbf{E} = \mathbf{E_1} \cup \mathbf{E_2}$

Step 5: $\mathbf{E_1} = (\mathbf{Q_1} \cap \mathbf{Q_2} \cap \mathbf{Q_3'}) \cup (\mathbf{Q_1} \cap \mathbf{Q_2'} \cap \mathbf{Q_3}) \cup (\mathbf{Q_1'} \cap \mathbf{Q_2} \cap \mathbf{Q_3})$

Step 6: $\mathbf{E_2} = \mathbf{Q_1} \cap \mathbf{Q_2} \cap \mathbf{Q_3}$

Step 7: $\mathbf{E} = \mathbf{E_1} \cup \mathbf{E_2} = (\mathbf{Q_1} \cap \mathbf{Q_2} \cap \mathbf{Q_3'}) \cup (\mathbf{Q_1} \cap \mathbf{Q_2'} \cap \mathbf{Q_3}) \cup (\mathbf{Q_1'} \cap \mathbf{Q_2} \cap \mathbf{Q_3}) \cup (\mathbf{Q_1} \cap \mathbf{Q_2} \cap \mathbf{Q_3})$

➤ d.
Let **E** = *exactly one jack is drawn.*

1.First card drawn is a jack and the others are not jacks: $\mathbf{J_1} \cap \mathbf{J_2'} \cap \mathbf{J_3'}$.

or

2.Second card drawn is a jack and the others are not jacks: $\mathbf{J_1'} \cap \mathbf{J_2} \cap \mathbf{J_3'}$.

or

3.Third card drawn is a jack and the others are not jacks: $\mathbf{J_1'} \cap \mathbf{J_2'} \cap \mathbf{J_3}$.

Therefore, $\mathbf{E} = (\mathbf{J_1} \cap \mathbf{J_2'} \cap \mathbf{J_3'}) \cup (\mathbf{J_1'} \cap \mathbf{J_2} \cap \mathbf{J_3'}) \cup (\mathbf{J_1'} \cap \mathbf{J_2'} \cap \mathbf{J_3})$

➤ e.
Let **E** = two cards drawn are Queens and one card is a Jack.

1.First card drawn is a jack and the others are queens: $\mathbf{J_1} \cap \mathbf{Q_2} \cap \mathbf{Q_3}$.

or

2.Second card drawn is a jack and the others are queens: $\mathbf{Q_1} \cap \mathbf{J_2} \cap \mathbf{Q_3}$.

or

3.Third card drawn is a jack and the others are queens: $\mathbf{Q_1 \cap Q_2 \cap J_3}$.

Therefore, $\mathbf{E} = (\mathbf{J_1 \cap Q_2 \cap Q_3}) \cup (\mathbf{Q_1 \cap J_2 \cap Q_3}) \cup (\mathbf{Q_1 \cap Q_2 \cap J_3})$

➤ f.
Let **E** = exactly two cards drawn are Queens

1.First card drawn is not a queen and the others are queens: $\mathbf{Q'_1 \cap Q_2 \cap Q_3}$.

or

2.Second card drawn is a not a queen and the others are queens: $\mathbf{Q_1 \cap Q_2' \cap Q_3}$.

or

3.Third card drawn is a not a queen and the others are queens: $\mathbf{Q_1 \cap Q_2 \cap Q'_3}$.

Therefore, $\mathbf{E} = (\mathbf{Q'_1 \cap Q_2 \cap Q_3}) \cup (\mathbf{Q_1 \cap Q'_2 \cap Q_3}) \cup (\mathbf{Q_1 \cap Q_2 \cap Q'_3})$

➤ g.
1.First card drawn is jack or king and the others are queens:

$(\mathbf{J_1 \cup K_1}) \cap \mathbf{Q_2 \cap Q_3}$.

or

2.Second card drawn is a jack or king and the others are queens:

$\mathbf{Q_1} \cap (\mathbf{J_2 \cup K_2}) \cap \mathbf{Q_3}$.

or

3.Third card drawn is a jack or king and the others are queens:

$\mathbf{Q_1 \cap Q_2} \cap (\mathbf{J_3 \cup K_3})$.

Therefore, $\mathbf{E} = [(\mathbf{J_1 \cup K_1}) \cap \mathbf{Q_2 \cap Q_3}] \cup [\mathbf{Q_1} \cap (\mathbf{J_2 \cup K_2}) \cap \mathbf{Q_3}] \cup [\mathbf{Q_1 \cap Q_2} \cap (\mathbf{J_3 \cup K_3})]$.

➤ h.
Let **E** *the first two cards drawn are kings.*

This event says nothing about the third card. Therefore $\mathbf{E} = \mathbf{K_1 \cap K_2}$

➤ i.
E = *no face cards are drawn.*

1. First card drawn is not a face card: $\mathbf{K_1}'\cap\mathbf{Q_1}'\cap\mathbf{J_3}'$

and

2. Second card drawn is not a face card: $\mathbf{K_2}'\cap\mathbf{Q_2}'\cap\mathbf{J_2}'$

and

3. Third card drawn is not a face card: $\mathbf{K_3}'\cap\mathbf{Q_3}'\cap\mathbf{J_3}'$

Therefore, $\mathbf{E} = (\mathbf{K_1}'\cap\mathbf{Q_1}'\cap\mathbf{J_3}')\cap(\mathbf{K_2}'\cap\mathbf{Q_2}'\cap\mathbf{J_2}')\cap(\mathbf{K_3}'\cap\mathbf{Q_3}'\cap\mathbf{J_3}')$.

If **F** is the event that a student will get financial aid, **J** is the event that he will find a part-time job,

and **G** is the event that he will graduate, express the following events:

15.
Let **E** = *A student who gets financial aid will also graduate or get a part-time job.*

E = *A student who gets financial aid and will also graduate or get a part-time job.*

Therefore, $\mathbf{E} = \mathbf{F}\cap(\mathbf{G}\cup\mathbf{J})$

16.
Let **E** = *A student who gets financial aid will not graduate and will not get a part-time job.*

E = *A student who gets financial aid and will not graduate and will not get a part-time job.*

Therefore, $\mathbf{E} = \mathbf{F}\cap\mathbf{G}'\cap\mathbf{J}'$

17.
Let **E** = A student will not graduate or will get a part-time job but will not get financial aid.

E = A student will not graduate or will get a part-time job and will not get financial aid.

Therefore, $\mathbf{E} = \mathbf{G}'\cup(\mathbf{J}\cap\mathbf{F}')$

A machine is producing ball bearings . Each hour, three ball bearings are selected at random, one at a time. Let $\mathbf{D_i}$(i = 1,2,3) represent the event that the ith ball bearing is defective. Write out the expression for the event **E:**

18.
E = *Exactly two ball bearings are defective.*

1. The first and second ball bearings are defective and the third is not:

$\mathbf{D_1 \cap D_2 \cap D_3'}$

or

2. The first and third ball bearings are defective and the second is not:

$\mathbf{D_1 \cap D_2' \cap D_3}$

or

3. The second and third ball bearings are defective and the first is not:

$\mathbf{D_1 \cap D_2 \cap D_3'}$

Therefore, $\mathbf{E} = (\mathbf{D_1 \cap D_2 \cap D_3'}) \cup (\mathbf{D_1 \cap D_2' \cap D_3}) \cup (\mathbf{D_1 \cap D_2' \cap D_3})$

19.
E = *at least one ball bearings is defective.*

Step 1: **E'** = *no ball bearings are defective.*

Step 2: $\mathbf{E'} = \mathbf{D_1' \cap D_2' \cap D_3'}$

Step 3: $\mathbf{E} = \mathbf{E''} = (\mathbf{D_1' \cap D_2' \cap D_3'})' = \mathbf{D_1'' \cup D_2'' \cup \cap D_3''} = \mathbf{D_1 \cup D_2 \cup \cap D_3}$

20.
E = *no ball bearings are defective.*

From problem 19, we have

$\mathbf{E} = \mathbf{D_1' \cap D_2' \cap D_3'}$

21.
E = *the first two drawn are defective and the last drawn is not defective.*

E = *the first drawn is defective <u>and</u> the second drawn is defective <u>and</u> the third drawn is not* *defective.*

Therefore, $\mathbf{E} = \mathbf{D_1} \cap \mathbf{D_2} \cap \mathbf{D_3}'$

Assume three cards are drawn from an ordinary deck of cards. If $\mathbf{K_i}$ (i = 1,2,3) are the events that the ith card drawn is a king and $\mathbf{Q_i}$ (i = 1,2,3) are the events that the ith card drawn is a queen. Write out the event **E** that

22.
E = *no king or queen is drawn.*

1. First card drawn is not a king and not a queen:

$\mathbf{K'_1 \cap Q'_1}$

and

2. The second card drawn is not a king and not a queen:

$\mathbf{K'_2 \cap Q'_2}$

and

3. The second card drawn is not a king and not a queen:

$\mathbf{K'_3 \cap Q'_3}$

Therefore, $\mathbf{E} = \mathbf{(K'_1 \cap Q'_1) \cap (K'_2 \cap Q'_2) \cap (K'_3 \cap Q'_3)}$

23.
E = *one card is a king and two are queens.*

1. First card drawn is a king and the other two are queens.

$\mathbf{K_1 \cap Q_2 \cap Q_3}$

or

2. Second card drawn is a king and the other two are queens.

$\mathbf{K_2 \cap Q_1 \cap Q_3}$

or

3. Second card drawn is a king and the other two are queens.

$\mathbf{K_3 \cap Q_1 \cap Q_2}$

Therefore, $\mathbf{E} = \mathbf{(K_1 \cap Q_2 \cap Q_3) \cup (K_2 \cap Q_1 \cap Q_3) \cup (K_3 \cap Q_1 \cap Q_2)}$

24.
E = *only one king and queen is selected.*

1. A king on the first and a queen on the second and neither on the third drawing:

$\mathbf{K_1 \cap Q_2 \cap K_1' \cap Q_3'}$

or

2. A king on the first and a queen on the third and neither on the second drawing:

$\mathbf{K_1 \cap Q_3 \cap K_2' \cap Q_2'}$

or

3. A queen on the first and a king on the second and neither on the third drawing:

$\mathbf{Q_1 \cap K_2 \cap K_3' \cap Q_3'}$

or

4. A queen on the first and a king on the third and neither on the second drawing:

$\mathbf{Q_1 \cap K_3 \cap K_2' \cap Q_2'}$

or

5. A queen on the second and a king on the third and neither on the first drawing:

$\mathbf{Q_2 \cap K_3 \cap K_1' \cap Q_1'}$

or

6. A queen on the third and a king on the second and neither on the first drawing:

$\mathbf{Q_3 \cap K_2 \cap K_1' \cap Q_1'}$

Therefore,

$\mathbf{E} = (\mathbf{K_1} \cap \mathbf{Q_2} \cap \mathbf{K_3}' \cap \mathbf{Q_3}') \cup (\mathbf{K_1} \cap \mathbf{Q_3} \cap \mathbf{K_2}' \cap \mathbf{Q_2}') \cup (\mathbf{Q_1} \cap \mathbf{K_2} \cap \mathbf{K_3}' \cap \mathbf{Q_3}') \cup$

$(\mathbf{Q_1} \cap \mathbf{K_3} \cap \mathbf{K_2}' \cap \mathbf{Q_2}') \cup (\mathbf{Q_2} \cap \mathbf{K_3} \cap \mathbf{K_1}' \cap \mathbf{Q_1}') \cup (\mathbf{Q_3} \cap \mathbf{K_2} \cap \mathbf{K_1}' \cap \mathbf{Q_1}')$

25.
Let **E** = *only one queen and no kings are selected.*

1. A queen is drawn on the first and no kings on the other drawings:

$\mathbf{Q_1} \cap \mathbf{K_1}' \cap \mathbf{K_2}'$

or

2. A queen is drawn on the second and no kings on the other drawings:

$(\mathbf{K_1}' \cap \mathbf{Q_2} \cap \mathbf{K_3}')$

or

3. A queen is drawn on the third and no kings on the other drawings:

$\mathbf{K_1}' \cap \mathbf{K_2}' \cap \mathbf{Q_3}$

Therefore, $\mathbf{E} = (\mathbf{Q_1} \cap \mathbf{K_1}' \cap \mathbf{K_2}') \cup (\mathbf{K_1}' \cap \mathbf{Q_2} \cap \mathbf{K_3}') \cup (\mathbf{K_1}' \cap \mathbf{K_2}' \cap \mathbf{Q_3})$

26.
1. A red was selected from urn 1 and a red was selected from urn 2:

$\mathbf{R_1} \cap \mathbf{R_2}$

or

2. A red was selected from urn 1 and a white was selected from urn 2:

$\mathbf{R_1}' \cap \mathbf{R_2}$

Therefore, $(\mathbf{R_1} \cap \mathbf{R_2}) \cup (\mathbf{R_1}' \cap \mathbf{R_2}) = \mathbf{R_2} \cap (\mathbf{R_1} \cup \mathbf{R_1}') = \mathbf{R_2} \cap \mathbf{S} = \mathbf{R_2}$.

A recent survey of men were taken to find out their participation in the following sports: football, baseball, and ice hockey. A member of this group is selected. Let **B** represent the event that he plays baseball, **F** represent the event that he plays football, and **I** represent the event that he plays ice hockey. Using these events along with unions, intersections and compliments, find the following events:

27.
Let **E** = *he only plays football and ice hockey*

E = *he plays football and ice hockey and does not play baseball.*

Therefore, **E** = **F∩I∩B'**

28.
Let **E** = *he only plays baseball.*

E = *he plays baseball and does not play football and does not place ice hockey.*

Therefore, **E** = **F'∩I'∩B.**

29.
Let **E** = *he only plays one of these sports.*

1. He plays baseball but does not play the other two sports:

B∩F'∩I'

or

2. He plays football but does not play the other two sports:

B'∩F∩I'

or

3. He plays ice hockey but does not play the other two sports:

(B'∩F'∩I)

Therefore, **E** = **(B∩F'∩I')∪(B'∩F∩I')∪(B'∩F'∩I)**

30.
Let **E** = *he does not play any of these sports.*

E = *he does not play a baseball <u>and</u> does not play football <u>and</u> does not play ice hockey.*

Therefore, **E** = **B'∩F'∩I'**

31.
$(\mathbf{F}\cap\mathbf{I}')$ = *he plays football and does not play ice hockey*

$(\mathbf{F}'\cap\mathbf{I})$ = *he does not plays football and he play ice hockey*

Therefore,

$(\mathbf{F}\cap\mathbf{I}')\cup(\mathbf{F}'\cap\mathbf{I})$ = *He plays football but not ice hockey or he plays ice hockey but not football.*

Using the laws on sets prove the following:

32.
$(\mathbf{A}\cup\mathbf{B})\cap(\mathbf{C}\cup\mathbf{D}) = [(\mathbf{A}\cup\mathbf{B})\cap\mathbf{C}]\cup[(\mathbf{A}\cup\mathbf{B})\cap\mathbf{D}] = [(\mathbf{A}\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{C})]\cup[(\mathbf{A}\cap\mathbf{D})\cup(\mathbf{B}\cap\mathbf{D})] =$

$[(\mathbf{A}\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{D})]\cup[(\mathbf{B}\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{D})]$

33.
➤ a.
Step 1: Assume $\mathbf{B}\cap\mathbf{A} = \phi$ and $\mathbf{B}\cup\mathbf{A} = \mathbf{S}$.

Step 2: $\mathbf{B} = \mathbf{B}\cup\phi = \mathbf{B}\cup(\mathbf{A}\cap\mathbf{A}') = (\mathbf{B}\cup\mathbf{A})\cap(\mathbf{B}\cup\mathbf{A}') = \mathbf{S}\cap(\mathbf{B}\cup\mathbf{A}') = \mathbf{B}\cup\mathbf{A}'$

Step 3: $\mathbf{A}' = \mathbf{A}'\cup\phi = \mathbf{A}'\cup(\mathbf{A}\cap\mathbf{B}) = (\mathbf{A}'\cup\mathbf{A})\cap(\mathbf{A}'\cup\mathbf{B}) = \mathbf{S}\cap(\mathbf{A}'\cup\mathbf{B}) = \mathbf{A}'\cup\mathbf{B} = \mathbf{B}\cup\mathbf{A}' = \mathbf{B}$.

➤ b.
$\mathbf{A}''$ is the compliment of $\mathbf{A}'$. But $\mathbf{A}$ is also the compliment of $\mathbf{A}'$ since they are compliments of each other. By the uniqueness of the compliments $\mathbf{A}'' = \mathbf{A}$.

➤ c.
$\mathbf{A}$ and $\mathbf{A}'$ are compliments of each other. We shall show that $\mathbf{A}\cap\mathbf{A}$ is the compliment of $\mathbf{A}'$.

Step 1: $(\mathbf{A}\cap\mathbf{A})\cap\mathbf{A}' = \mathbf{A}\cap(\mathbf{A}\cap\mathbf{A}') = \mathbf{A}\cap\phi = \phi$.

Step 2: $(\mathbf{A}\cap\mathbf{A})\cup\mathbf{A}' = (\mathbf{A}\cup\mathbf{A}')\cap(\mathbf{A}\cup\mathbf{A}') = \mathbf{S}\cap\mathbf{S} = \mathbf{S}$.

step 3: Therefore, $\mathbf{A}\cap\mathbf{A}$ is the compliment of $\mathbf{A}'$ and from the uniqueness of the compliment, $\mathbf{A}\cap\mathbf{A} = \mathbf{A}$.

➤ d.
$\mathbf{A} = \mathbf{A}\cup\phi = \mathbf{A}\cup(\mathbf{A}\cap\mathbf{A}') = (\mathbf{A}\cup\mathbf{A})\cap(\mathbf{A}\cup\mathbf{A}') = (\mathbf{A}\cup\mathbf{A})\cap\mathbf{S} = \mathbf{A}\cup\mathbf{A}$

➤ e.

Step 1: $\mathbf{S} = \mathbf{A} \cup \mathbf{A'}$

Step 2: $\mathbf{S} \cup \mathbf{A} = (\mathbf{A} \cup \mathbf{A'}) \cup \mathbf{A} = (\mathbf{A'} \cup \mathbf{A}) \cup \mathbf{A} = \mathbf{A'} \cup (\mathbf{A} \cup \mathbf{A}) = \mathbf{A'} \cup \mathbf{A} = \mathbf{S}$

➤ f.

Step 1: $\phi = \mathbf{A} \cap \mathbf{A'}$

Step 2: $\mathbf{A} \cap \phi = \mathbf{A} \cap (\mathbf{A} \cap \mathbf{A'}) = (\mathbf{A} \cap \mathbf{A}) \cap \mathbf{A'} = \mathbf{A} \cap \mathbf{A'} = \phi$

➤ g.

ϕ' is the compliment of ϕ. We shall show that **S** is the compliment of ϕ.

Step 1: $\mathbf{S} \cap \phi = \phi$

Step 2: $\mathbf{S} \cup \phi = \mathbf{S}$

Therefore, **S** is the compliment of ϕ: $\mathbf{S} = \phi$.

➤ h.

Step 1: From (c.) we know $\phi' = \mathbf{S}$.

Step 2: From (b.) we know $\mathbf{S'} = \phi$.

➤ i.

The compliment of $\mathbf{A} \cup \mathbf{B}$ is $(\mathbf{A} \cup \mathbf{B})'$. We shall show $\mathbf{A'} \cap \mathbf{B'}$ is equal to the compliment of $\mathbf{A} \cup \mathbf{B}$.

Step 1: $(\mathbf{A'} \cap \mathbf{B'}) \cap (\mathbf{A} \cup \mathbf{B}) = [(\mathbf{A'} \cap \mathbf{B'}) \cap \mathbf{A}] \cup [(\mathbf{A'} \cap \mathbf{B'}) \cap \mathbf{B}] = \phi \cup \phi = \phi$.

Step 2: $(\mathbf{A'} \cap \mathbf{B'}) \cup (\mathbf{A} \cup \mathbf{B}) = [\mathbf{A'} \cup (\mathbf{A} \cup \mathbf{B})] \cap [\mathbf{B'} \cup (\mathbf{A} \cup \mathbf{B})] = [(\mathbf{A'} \cup \mathbf{A}) \cup \mathbf{B}] \cap [(\mathbf{B'} \cup \mathbf{B}) \cup \mathbf{A})] =$

$(\mathbf{S} \cup \mathbf{B}) \cap (\mathbf{S} \cup \mathbf{A}) = (\mathbf{S} \cap \mathbf{S}) = \mathbf{S}$

Step 3: Therefore, $\mathbf{A'} \cap \mathbf{B'}$ is the compliment of $\mathbf{A} \cup \mathbf{B}$ and from uniqueness of the compliment, $(\mathbf{A} \cup \mathbf{B})' = \mathbf{A'} \cap \mathbf{B'}$

$(\mathbf{A} \cap \mathbf{B})' = \mathbf{A'} \cup \mathbf{B'}$

The compliment of $\mathbf{A} \cap \mathbf{B}$ is $(\mathbf{A} \cap \mathbf{B})'$. We shall show $\mathbf{A'} \cup \mathbf{B'}$ is equal to the compliment of $\mathbf{A} \cap \mathbf{B}$.

Step 1: $(\mathbf{A'} \cup \mathbf{B'}) \cap (\mathbf{A} \cap \mathbf{B}) = [(\mathbf{A'} \cap (\mathbf{A} \cap \mathbf{B})] \cup [(\mathbf{B'} \cap (\mathbf{A} \cap \mathbf{B})] = \phi \cup \phi = \phi$.

Step 2: $(\mathbf{A}'\cup\mathbf{B}')\cup(\mathbf{A}\cap\mathbf{B}) = [(\mathbf{A}'\cup\mathbf{A})\cup\mathbf{B}']\cap[\mathbf{A}'\cup(\mathbf{B}'\cup\mathbf{B})] = [\mathbf{S}\cup\mathbf{B}']\cap[\mathbf{A}'\cup\mathbf{S}] = \mathbf{S}\cap\mathbf{S} = \mathbf{S}$

Step 3: Therefore, $(\mathbf{A}'\cup\mathbf{B}')$ is the compliment of $(\mathbf{A}\cap\mathbf{B})$ and from uniqueness of the compliment,

$'(\mathbf{A}'\cup\mathbf{B}') = (\mathbf{A}\cap\mathbf{B})'$.

➤ j.
Step 1: Since $\mathbf{A} \subseteq \mathbf{B}$ then $\mathbf{A}\cap\mathbf{B}' = \phi$.

Step 2: Since $\mathbf{B} \subseteq \mathbf{A}$ then $\mathbf{B}\cap\mathbf{A}' = \phi$.

Step 3: $\mathbf{A} = \mathbf{A}\cap(\mathbf{B}\cup\mathbf{B}') = (\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{B}') = (\mathbf{A}\cap\mathbf{B})\cup\phi = \mathbf{A}\cap\mathbf{B}$

Step 4: $\mathbf{B} = \mathbf{B}\cap(\mathbf{A}\cup\mathbf{A}') = (\mathbf{A}\cap\mathbf{B})\cup(\mathbf{B}\cap\mathbf{A}') = (\mathbf{A}\cap\mathbf{B})\cup\phi = \mathbf{A}\cap\mathbf{B}$

Step 5: Therefore, $\mathbf{A} = \mathbf{B}$.

➤ k.
Step 1: Since $\mathbf{A} \subseteq \mathbf{B}$ then $\mathbf{A}\cap\mathbf{B}' = \phi$.

Step 2: $\mathbf{B}'\cap(\mathbf{B}\cup\mathbf{A}) = (\mathbf{B}'\cap\mathbf{B})\cup(\mathbf{B}'\cap\mathbf{A}) = \phi$

Therefore, $(\mathbf{B}\cup\mathbf{A}) \subseteq \mathbf{B}$.

Step 3: $\mathbf{B}\cap(\mathbf{B}\cup\mathbf{A})' = \mathbf{B}\cap(\mathbf{B}'\cap\mathbf{A}') = \phi$

Therefore, $\mathbf{B} \subseteq (\mathbf{B}\cup\mathbf{A})$

Step 4: Since $(\mathbf{B}\cup\mathbf{A}) \subseteq \mathbf{B}$ and $\mathbf{B} \subseteq (\mathbf{B}\cup\mathbf{A})$ then $\mathbf{B} = \mathbf{B}\cup\mathbf{A}$**.**

➤ l.
Step 1: Since $\mathbf{A} \subseteq \mathbf{B}$ then $\mathbf{A}\cap\mathbf{B}' = \phi$.

Step 2: $\mathbf{A}\cap(\mathbf{B}\cap\mathbf{A})' = \mathbf{A}\cap(\mathbf{B}'\cup\mathbf{A}') = (\mathbf{A}\cap\mathbf{B}')\cup(\mathbf{A}\cap\mathbf{A}') = \phi\cup\phi = \phi$

Therefore, $\mathbf{A} \subseteq \mathbf{B}\cap\mathbf{A}$.

Step 3: $\mathbf{A}'\cap(\mathbf{B}\cap\mathbf{A}) = \phi$

Therefore, $\mathbf{B}\cap\mathbf{A} \subseteq \mathbf{A}$

Step 4: Since $(\mathbf{B}\cap\mathbf{A}) \subseteq \mathbf{A}$ and $\mathbf{A} \subseteq (\mathbf{B}\cap\mathbf{A})$ then $\mathbf{B} = \mathbf{B}\cap\mathbf{A}$**.**

➤ m.
Step 1: Since **A** ⊆ **B** then **A**∩**B**'= ϕ.

Step 2: Since **B** ⊆ **C** then **B**∩**C**'= ϕ and **C** = **B**∪**C**.

Step 3: **A**∩**C**' = **A**∩(**B**∪**C**)' = **A**∩(**B**'∩**C**') = (**A**∩**B**')∩**C**' = ϕ∩**C**' = ϕ

Therefore, **A** ⊆ **C.**

➤ n.
Step 1: Since **A** ⊆ **B** then **A**∩**B**' = ϕ

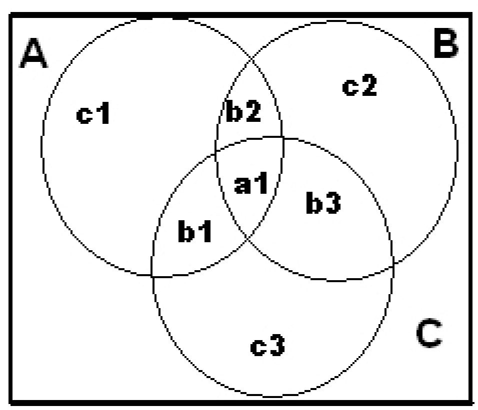
Step 2: Let **C** = **B**'. Then **A**∩**C** =**A**∩**B**' = ϕ.

Step 3: Since **C**∩**A** = ϕ then **C** ⊆ **A'.**

Step 4: **B' = C** ⊆ **A**'.

➤ o.
Since ϕ∩**A**' = ϕ then ϕ ⊆ **A.**

34.
➤ a.
From the venn diagram we have the following



a1: **A**∩**B**∩**C**

b1: (**A**∩**C**)∩**B**′

b2: (**A**∩**B**)∩**C**′

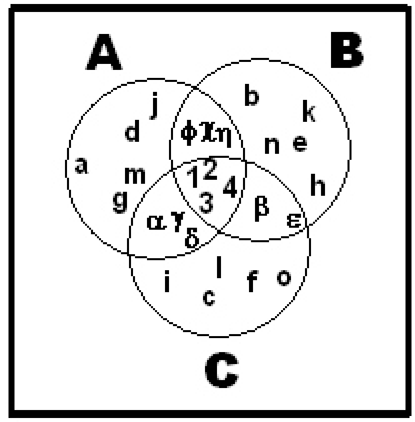
b3: **A**′∩(**B**∩**C**)

c1: **A**∩(**B**′∩**C**′)

c2: (**A**′∩**C**′)∩**B**

c3: (**A**′∩**B**′)∩**C**

➤ b.
Representing the union as a Venn diagram gives us



$\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$: {1,2,3,4}

$(\mathbf{A} \cap \mathbf{C}) \cap \mathbf{B}'$: {

$(\mathbf{A} \cap \mathbf{B}) \cap \mathbf{C}'$:{

$\mathbf{A}' \cap (\mathbf{B} \cap \mathbf{C})$: {

$\mathbf{A} \cap (\mathbf{B}' \cap \mathbf{C}')$: {a,g,m,d,j}

$(\mathbf{A}' \cap \mathbf{C}') \cap \mathbf{B}$: {b,n,e,k,h}

$(\mathbf{A}' \cap \mathbf{B}') \cap \mathbf{C}$: {i,c,l,f,o}

Associative law: $(\mathbf{B} \cap \mathbf{C}) \cup (\mathbf{B} \cap \mathbf{C}') \cup (\mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{B}' \cap \mathbf{C}') = [(\mathbf{B} \cap \mathbf{C}) \cup (\mathbf{B} \cap \mathbf{C}')] \cup [(\mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{B}' \cap \mathbf{C}')]$

Distributive law: $[(\mathbf{B} \cap \mathbf{C}) \cup (\mathbf{B} \cap \mathbf{C}')] \cup [(\mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{B}' \cap \mathbf{C}')]$ $[\mathbf{B} \cap (\mathbf{C} \cup \mathbf{C}')] \cup [(\mathbf{B}' \cap (\mathbf{C} \cup \mathbf{C}')]$

$[\mathbf{B} \cap (\mathbf{C} \cup \mathbf{C}')] \cup [(\mathbf{B}' \cap (\mathbf{C} \cup \mathbf{C}')] = (\mathbf{B} \cap \mathbf{S}) \cup (\mathbf{B}' \cap \mathbf{S}) = \mathbf{B} \cup \mathbf{B}' = \mathbf{S}$

36.
Distributive law: $(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) \cup (\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}') \cup (\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}') =$

$\mathbf{A} \cap [(\mathbf{B} \cap \mathbf{C}) \cup (\mathbf{B} \cap \mathbf{C}') \cup (\mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{B}' \cap \mathbf{C}')]$

From problem 35, $\mathbf{A} \cap [(\mathbf{B} \cap \mathbf{C}) \cup (\mathbf{B} \cap \mathbf{C}') \cup (\mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{B}' \cap \mathbf{C}')] = \mathbf{A} \cap \mathbf{S} = \mathbf{A}$

37.
$\mathbf{A} \cap \mathbf{B}' = \phi$

$(\mathbf{B} \cup \mathbf{C})' = \mathbf{B}' \cap \mathbf{C}'$

$\mathbf{A} \cap (\mathbf{B} \cup \mathbf{C})' = \mathbf{A} \cap (\mathbf{B}' \cap \mathbf{C}') = (\mathbf{A} \cap \mathbf{B}') \cap \mathbf{C}' = \phi \cap \mathbf{C}' = \phi$

Assume **A** is not a subset of **B**. Then $\mathbf{A} \cap \mathbf{B}' \neq \phi$

$\mathbf{A} = (\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{A} \cap \mathbf{B}')$

$(\mathbf{A} \cap \mathbf{B}')$ is a proper subset of **A**. Assume otherwise: $\mathbf{A} = \mathbf{A} \cap \mathbf{B}'$. This gives us

$\mathbf{A}\cap\mathbf{B} = \mathbf{A}\cap\mathbf{B}'\cap\mathbf{B} = \phi$. This means **A** is a subset of $\mathbf{B}'$. But this is not possible since all proper subsets are subsets of **B**.

Since $(\mathbf{A}\cap\mathbf{B}')$ is a proper subset of **A** it is also a subset of **B**. But $(\mathbf{A}\cap\mathbf{B}')$ is a subset of $\mathbf{B}'$. But this can only happen if $(\mathbf{A}\cap\mathbf{B}') = \phi$.

39.
Step 1: Since **S** is the sample space, $\mathbf{E} \subseteq \mathbf{S}$.

Step 2: $\mathbf{E} = (\mathbf{K}_1 \cap\mathbf{K}_2\cap\mathbf{K}_3)\cup(\mathbf{K}_1{}'\cap\mathbf{K}_2\cap\mathbf{K}_3)\cup(\mathbf{K}_1 \cap\mathbf{K}_2{}'\cap\mathbf{K}_3)\cup(\mathbf{K}_1 \cap\mathbf{K}_2\cap\mathbf{K}_3{}')\cup$

$(\mathbf{K}_1{}'\cap\mathbf{K}_2{}'\cap\mathbf{K}_3)\cup(\mathbf{K}_1{}'\cap\mathbf{K}_2\cap\mathbf{K}_3{}')\cup(\mathbf{K}_1 \cap\mathbf{K}_2{}'\cap\mathbf{K}_3{}')\cup(\mathbf{K}_1{}' \cap\mathbf{K}_2{}'\cap\mathbf{K}_3{}')$

Let **A** be an arbitrary event in **S**. We know one of the following is true of **A:**

$\mathbf{A} \subseteq (\mathbf{K}_1{}' \cap\mathbf{K}_2{}'\cap\mathbf{K}_3{}')$, the event no kings are drawn.
or
$\mathbf{A} \subseteq (\mathbf{K}_1{}'\cap\mathbf{K}_2{}'\cap\mathbf{K}_3)\cup(\mathbf{K}_1{}'\cap\mathbf{K}_2\cap\mathbf{K}_3{}')\cup(\mathbf{K}_1 \cap\mathbf{K}_2{}'\cap\mathbf{K}_3{}')$, the event exactly 1 king is drawn.
or
$\mathbf{A}: \subseteq (\mathbf{K}_1{}'\cap\mathbf{K}_2\cap\mathbf{K}_3)\cup(\mathbf{K}_1 \cap\mathbf{K}_2{}'\cap\mathbf{K}_3)\cup (\mathbf{K}_1 \cap\mathbf{K}_2\cap\mathbf{K}_3{}')$, the event exactly 2 kings are drawn.
or
$\mathbf{A} = (\mathbf{K}_1 \cap\mathbf{K}_2\cap\mathbf{K}_3)$, The event exactly 3 kings are drawn.

From problem 37, therefore, $\mathbf{A} \subseteq \mathbf{E}$.

From problem 38, since all the subsets of **S** are in **E**, then $\mathbf{S} \subseteq \mathbf{E}$.

By the anti-symmetric law, $\mathbf{E} = \mathbf{S}$.

40. Assume there is a non-empty set **E** that is a member of this family of sets.
Since it is always true that $\phi \subseteq \mathbf{E}$, then $\mathbf{E} \subseteq \phi$. Therefore, by the anti-symmetry law, $\mathbf{E} = \phi$.