

# Set Theory

## Lesson 8

### Experiments, Sample Spaces and Events

#### 8.1- What is an Experiment?

**8.1 Problem 1:**

Since the customer is only purchasing one of these cars, the possible single outcomes are Ford, Honda, Datsun.

**8.1 - Problem 2:**

Since only one member of the stamp club is selected, the possible single outcomes are Mary, Jane, Bill, Henry, Gayle, Frank.

**8.1 - Problem 3:**

Here we list all possible phone call each day. Each day she makes between 20 and thirty call: 20,21...,30

**8.1 - Problem 4:**

A coin tossed three times, produces three possibilities, heads and tails. Letting h stand for heads and t for tail, the list is all possible results from tossing a coin three times in their natural order:

(h,h,h),(h,h,t),(h,t,h),(t,h,h),(t,t,h),(t,h,t),(h,t,t),(t,t,t).

For example, (t,h,t) means that the first toss resulted in tails, the second toss resulted in heads and the third toss resulted in tails.

#### 8.2 - What is a Sample Space?

**8.2 - Problem 1:**

The sample space is the universal space created by the list of single outcomes: {Ford,Honda,Datsun}

(See 8.1 - Problem 1).

**8.2 - Problem 2:**

The sample space is the universal space created by the list of single outcomes:

$S = \{Mary,Jane,Bill,Henry,Gayle,Frank\}$  (see Problem 1.2).

**8.2 - Problem 3:**

The sample space is the universal space created by the list of single outcomes:  $S = \{20,21...,30\}$ . (see Problem 1.3).

**8.2 - Problem 4:**

The sample space is the universal space created by the list of single outcomes:

$$S = \{(h,h,h),(h,h,t),(h,t,h),(t,h,h),(t,t,h),(t,h,t),(h,t,t),(t,t,t)\}. \text{ (see Problem 1.4).}$$

**8.3 - What is an Event?****8.3 - Problem 1:**

Step 1: The sample space is  $S = \{1,2,3,4,5,6\}$ .

Step 2: The event that the numbers are greater than three is  $E = \{4,5,6\}$ .

**8.3 - Problem 2:**

Step 1: The sample space is  $S = \{(h,h),(h,t),(t,h),(t,t)\}$ .

Step 2: The event that the a head must occur on the first toss allows heads or tails on the second toss. Therefore,  $E = \{(h,h),(h,t)\}$ .

**8.3 - Problem 3:**

The sample space is  $S = \{0,1,2,3,4,5\}$ .

►(a).

The event she receives more than 1 call a days means she receives between two and five calls a day:  
 $E = \{2,3,4,5\}$

►(b).

The event she receives at least 4 calls a day means that she receives between 4 and five calls a day:  
 $E = \{4,5\}$ .

►(c).

The event she receives no calls a day is  $E = \{0\}$ .

**8.3 - Problem 4:**

The sample space is

$$S = \{(h,h,h),(h,t,t),(t,h,t),(t,t,h),(h,h,t),(h,t,h),(t,h,h),(t,t,t)\}.$$

►(a).

This event occurs when only three heads or three tails occur:

$$E = \{(h,h,h),(t,t,t)\}$$

►(b).

This event occurs when one heads occur on any toss and the other two are tails:

$$\mathbf{E} = \{(h,t,t),(t,h,t),(t,t,h)\}$$

►(c).

This event occurs when two heads occur on any tosses and the other toss results in tails:

$$\mathbf{E} = \{(h,h,t),(h,t,h),(t,h,h)\}.$$

►(d).

This event occurs when two or three tails occur on any of the three tosses:

$$\mathbf{E} = \{(h,t,t),(t,h,t),(t,t,h),(t,t,t)\}$$

►(e).

This event occurs when 0,1,2 tails occur on any of the three tosses:

$$\mathbf{E} = \{(t,t,h),(t,h,t),(h,t,t),(h,h,t),(h,t,h),(t,h,h),(h,h,h)\}$$

### Supplementary Problems.

1.

The experiment is as follows: toss the coin until a tail occurs and record each toss. Therefore, the sample space is

$$\mathbf{S} = \{(t),(h,t),(h,h,t),\dots,(h,h,h,\dots,h,t)\}$$

For example  $(h,h,h,t)$  means the first three tosses resulted in heads and the fourth toss is tails.

2.

At most four tosses means four tosses or less. Therefore, the event is  $\mathbf{E} = \{(t),(h,t),(h,h,t),(h,h,h,t)\}$ .

3.

The event that it took at least three tosses means that three or more tosses occurred. Therefore, the event is  $\mathbf{E} = \{(h,h,t),(h,h,h,t),(h,h,h,h,t),\dots\}$ .

4.

The event that it took between two and four tosses (inclusive) means two, three or four tosses occurred. This event is  $\mathbf{E} = \{(h,t),(h,h,t),(h,h,h,t)\}$ .

A fair coin is tossed until two heads occur or tossed four times, whichever occurs first.

**5.**

We stop tossing as soon as two heads appear or four tosses, whichever occurs first.

Two tosses: (h,h)

Three tosses: (t,h,h),(h,t,h)

four tosses: (t,t,h,h),(t,h,t,h),(h,t,t,h), (h,t,t,t),(t,h,t,t),(t,t,h,t),(t,t,t,h),(t,t,t,t)

Combining these together gives:

$$S = \{(h,h),(t,h,h),(h,t,h),(t,t,h,h),(t,h,t,h),(h,t,t,h), (h,t,t,t),(t,h,t,t),(t,t,h,t),(t,t,t,h),(t,t,t,t)\}$$
**6.**

Since the sample space  $S$  requires at least 2 tosses of the coin, then the event is number of tosses between two and three tosses:

two tosses: (h,h)

three tosses: (t,h,h),(h,t,h)

Combining these we get  $E = \{(h,h),(t,h,h),(h,t,h)\}$

**7.**

The event,  $E$ , more than three tosses were made is equal to four or more. But since the sample space  $S$  allows a maximum four tosses, this event is equal to exactly four tosses occurred. Therefore,

$$E = \{(t,t,h,h),(t,h,t,h),(h,t,t,h),(h,t,t,t),(t,h,t,t),(t,t,h,t),(t,t,t,h),(t,t,t,t)\}$$
**8.**

Since heads must occur on the first toss, we take from the sample space  $S$  all elements where h appears in the first position:

$$E = \{(h,h),(h,t,h),(h,t,t,h), (h,t,t,t)\}$$

Mr. Jones tosses a pair of dice once.

**9.**

This event is equal to tossing a sum 8 or more. The maximum sum possible is  $6 + 6 = 12$ . The following are all possible ways this happens:

eight: (2,6),(3,5),(4,4),(5,3),(6,2)

nine: (3,6),(4,5),(5,4),(6,3)

ten:  $(4,6),(5,5),(6,4)$

eleven:  $(5,6),(6,5)$

twelve:  $(6,6)$ .

Note for example that  $(6,3)$  is different than  $(3,6)$ .

Combining these together gives the required event:

$$E = \{(2,6),(3,5),(4,4),(5,3),(6,2),(3,6),(4,5),(5,4),(6,3),(4,6),(5,5),(6,4),(5,6),(6,5),(6,6)\}.$$

**10.**

The event that he tosses a sum less than 4 is equal to the sum between 2 and 3. This event is

$$E = \{(1,1),(1,2),(2,1)\}.$$

**11.**

The event that he tosses a sum between 7 and 9 (inclusive) follows from

seven:  $(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)$

eight:  $(3,5),(4,4),(5,3),(6,2)$

nine:  $(3,6),(4,5),(5,4),(6,3)$

$$E = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(3,5),(4,4),(5,3),(6,2),(3,6),(4,5),(5,4),(6,3)\}$$

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