SET THEORY
Lesson 7
Cardinality of a Set

7.1- What is the Cardinality of a Set?

7.1 - Problem 1:
The elements of the set **A** are the letters of the alphabet: A = {a,b,c,d,e,f,g,...,w,x,y,z}. Since there are 26 letters in the alphabet, the cardinality of **A** is 26: **#A** = 26.

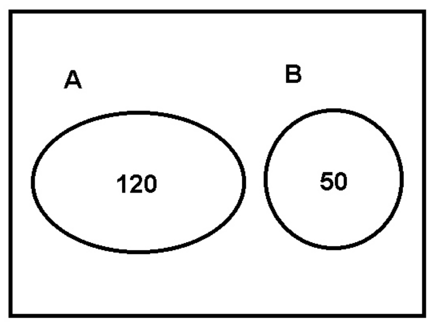
7.1 - Problem 2:
Each element of this set a group of 4 letters (h,t). For example (h,h,t,h) is a single element of the set **F.** Since there are 8 different groups (elements) that make of this set **F**, the cardinality of **F** is 8: #**F** = 8.

7.1 - Problem 3:
Step 1: **K**∪**W** = {1,2,3,4,5}∪{6,7,8,9,10} = {1,2,3,4,5,6,7,8,9,10}

Step 2: (**K**∪**W**)'= {1,2,3,4,5,6,7,8,9,10}' = {0}

Step 3: (**K**∪**W**)′∩ **T** = ∩**T** = ϕ

Step 4: #[(**K**∪**W**)′∩ **T**] = # ϕ = 0.



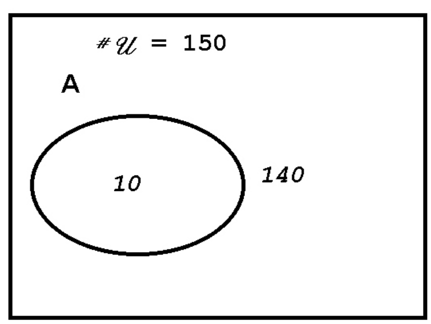
7.2 - Rules on Cardinality of Sets

7.2 - Problem 1:
Step 1: Using Rule 1 we have

#(**A**∪**B**) = #**A** + **#B - #(A**∩**B**)

Step 2: since #(**A**∩**B**) = 0 then

#(**A**∪**B**) = #**A** + **#B** = 120 + 50 = 170

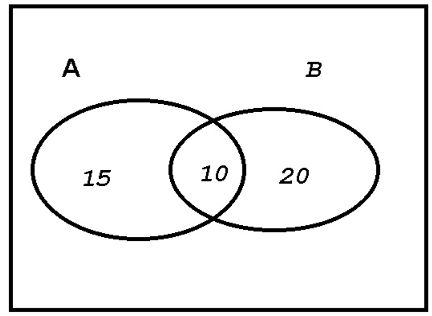


7.2 - Problem 2:
Rule 2: #(**A'**∩**B**) = #**B** - #(**A**∩**B**) = 55 - 0 = 55.

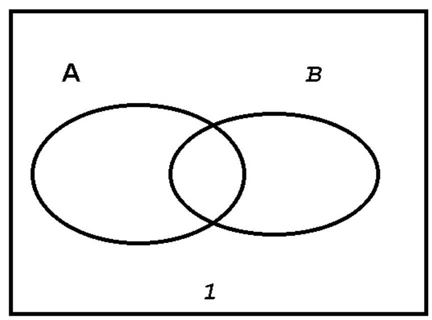
7.2 - Problem 3:
Rule 3: #**A**′= #𝒰 - #**A** = 150 - 10 = 140.

7.2 - Problem 4:
Rule 4:

#(**A**∪**B**) = #(**A**∩**B**′) + #(**A**′∩**B**) + #(**A**∩**B**) = 15 + 20+ 10 = 45.



7.2 - Problem 5:
Rule 5: #[(**A**∪**B**)′] = #(**A**′∩**B**′) = 1.



7.2 - Problem 6:
➤(a). Rule 2: #(**A**′∩**B**) = #**B** - #(**A** ∩ **B**) = 5 - 0 = 0.

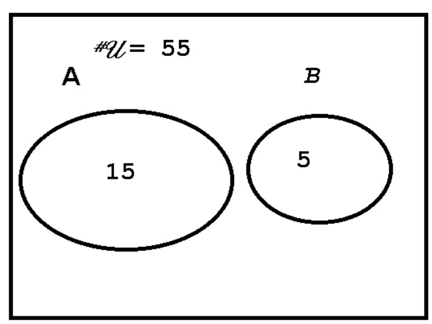
➤(b). Rule 2: #(**A**′∩**B**) = #**A** - #(**A** ∩ **B**) = 15 - 0 = 15.

➤(c). Rule 1: #(**A**∪**B**) = #**A** + #**B** - #(**A** ∩ **B**) = 15 + 5 - 0 = 20.

➤(d).
Step 1: Since #(**A**∩**B**) = 0 we have **A**∩**B** = ϕ.

Step 2: #(**A**∩**B**)′ = #(ϕ)' = #𝒰 = 55.

➤(e).
Rule 3: #**A**′ = #𝒰 - #**A** = **55** - 15 = 40.



7.2 - Problem 7:

➤(a).
#𝒰= 35 + 40 + 50 + 20 = 145.

➤(b).
step 1: Rule 2: #(**A**∩**B**') = #**A** - #(**A** ∩ **B**)

Step 2: #**A** = #(**A**∩**B**') + #(**A**∩**B**) = 40 + 50 = 90.

➤(c).
step 1: Rule 2: #(**A**∩**B**') = #**A** - #(**A** ∩ **B**)

Step 2: #**B** = #(**A**'∩**B**) + #(**A**∩**B**) = 20 + 50 = 70.

➤(d).
Rule 2: #(**A**′∩**B**) = #**B** - #(**A** ∩ **B**) = 50 + 20 - 50 = 20.

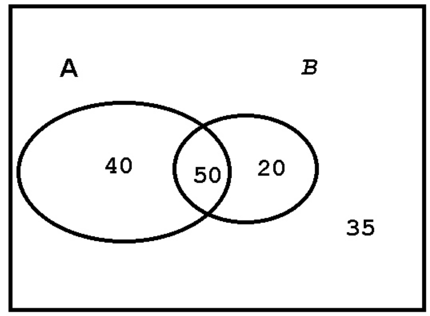
➤(e).
Rule 2: #(**A**∩**B**') = #**A** - #(**A** ∩ **B**) = 50 + 40 - 50 = 40.

➤(f).
Rule 3: $\#(\mathbf{A}\cap\mathbf{B})' = \#\mathcal{U} - \#(\mathbf{A}\cap\mathbf{B}) = 145 - 50 = 95$

➤(g).
Step 1: Rule 5: $\#(\mathbf{A}'\cap\mathbf{B}') = \#[(\mathbf{A}\cup\mathbf{B})']$

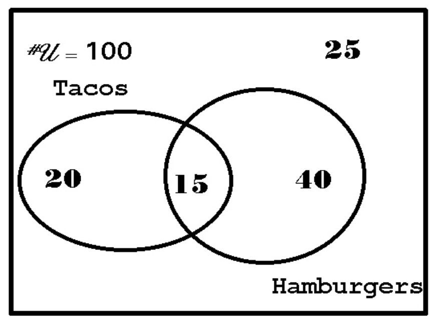
Step 2: Rule 3: $\#[(\mathbf{A}\cup\mathbf{B})'] = \#\mathcal{U} - \#(\mathbf{A}\cup\mathbf{B}) =$

$145 - (40 + 50 + 20) = 35$



7.3 - Counting Applications.

7.3 - Problem 1:
The Venn diagram on the right represents the division of customers that eat tacos and hamburgers.



Let **T** represent those customers that ordered tacos and **H** those customers that ordered hamburgers.

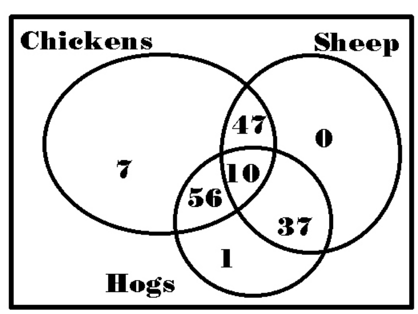
➤ a. The event "ordered only tacos" = $\mathbf{T}\cap\mathbf{H}'$
Therefore, the number of customers that ordered only tacos is $\#(\mathbf{T}\cap\mathbf{H}') = 20$.

➤ b.
The event "ordered only hamburgers" = $\mathbf{T}'\cap\mathbf{H}$
Therefore, the number of customers that ordered only hamburgers is $\#(\mathbf{T}'\cap\mathbf{H}) = 40$.

➤ c.
The event that the customer ordered "at least one of these food items" = $\mathbf{T}\cup\mathbf{H}$. Therefore, the number of customers that ordered at least one of these food items is $\#(\mathbf{T}\cup\mathbf{H}) = 20 + 15 + 40 = 75$.

➤ d.
The event that the customer "ordered neither tacos nor hamburgers" = $\mathbf{T}'\cap\mathbf{H}' = (\mathbf{T}\cup\mathbf{H})'$. Therefore, the number of customers that ordered neither of these food items is $\#(\mathbf{T}\cup\mathbf{H})' = 25$.

7.3 - Problem 2:
Let **C** = Chickens
Let **S** = Sheep
Let **H** = hogs



➤(a).
The event "raised only one type of these animals".

Step 1: $\mathbf{E} = [\mathbf{C} \cap (\mathbf{S} \cup \mathbf{H})'] \cup [\mathbf{S} \cap (\mathbf{C} \cup \mathbf{H})'] \cup [\mathbf{H} \cap (\mathbf{C} \cup \mathbf{S})']$

Therefore,

Step 2: $\#\mathbf{E} = \#[\mathbf{C} \cap (\mathbf{S} \cup \mathbf{H})'] + \#[\mathbf{S} \cap (\mathbf{C} \cup \mathbf{H})'] + \#[\mathbf{H} \cap (\mathbf{C} \cup \mathbf{S})'] = 7 + 0 + 1 = 8$

➤(b).
Step 1: Let **E** = "raised at least two types of these animals" is "raised exactly two or raised all three".

Step 2: The event "raised exactly two" is $[\mathbf{C} \cap \mathbf{S} \cap \mathbf{H}'] \cup [\mathbf{C} \cap \mathbf{S}' \cap \mathbf{H}] \cup [\mathbf{C}' \cap \mathbf{S} \cap \mathbf{H}]$

Step 3: The event "raised all three" is $\mathbf{C} \cap \mathbf{S} \cap \mathbf{H}$.

Step 4: $\mathbf{E} = [\mathbf{C} \cap \mathbf{S} \cap \mathbf{H}'] \cup [\mathbf{C} \cap \mathbf{S}' \cap \mathbf{H}] \cup [\mathbf{C}' \cap \mathbf{S} \cap \mathbf{H}] \cup [\mathbf{C} \cap \mathbf{S} \cap \mathbf{H}]$

Step 5: $\#\mathbf{E} = \#[\mathbf{C} \cap \mathbf{S} \cap \mathbf{H}'] + \#[\mathbf{C} \cap \mathbf{S}' \cap \mathbf{H}] + \#[\mathbf{C}' \cap \mathbf{S} \cap \mathbf{H}] + \#[\mathbf{C} \cap \mathbf{S} \cap \mathbf{H}] = 47 + 56 + 37 + 10 = 150$.

➤(c).
Step 1: The event **E** = "raised no sheep" = $\mathbf{S}'$.

Step 2: $\#\mathbf{E} = \#\mathbf{S}' = 7 + 56 + 1 = 64$.

➤(d).
Step 1: The event **E** = "raised sheep or hogs" = $\mathbf{S} \cup \mathbf{H}$.

Step 2: $\#\mathbf{E} = \#(\mathbf{S} \cup \mathbf{H}) = \#\mathbf{S} + \#\mathbf{H} - \#(\mathbf{S} \cap \mathbf{H}) = 94 + 104 - 47 = 151$.

➤(e).
Step 1: The event **E** = "raised sheep or hogs but not chickens" $= (\mathbf{S} \cup \mathbf{H}) \cap \mathbf{C}'$ $1 + 37 + 0 = 38$.

➤(f).
Step 1: The event **E** = "raised none of these animals." $= (\mathbf{S} \cup \mathbf{H} \cup \mathbf{C})'$

Step 2: $\#\mathbf{E} = \#(\mathbf{S} \cup \mathbf{H} \cup \mathbf{C})'$

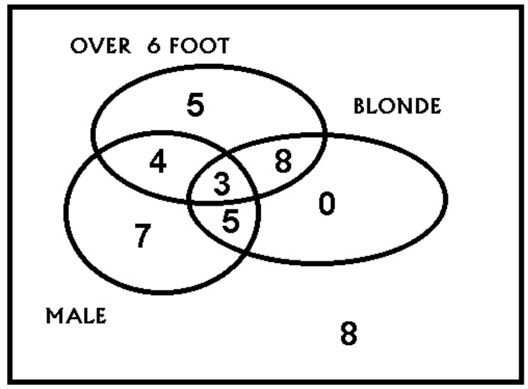
Since, we don't know the cardinality of the universal set, we can't determine #**E**.

➤(g).
Step 1: The event **E** = "raised at lest one of these animals." $= \mathbf{S} \cup \mathbf{H} \cup \mathbf{C}$.

Step 2: **#E** = #(**S**∪**H**∪**C**) = 7 + 56 + 10 + 47 + 1 + 37 + 0 = 158

Supplementary Problems

1.
From the diagram, we count the number of students in the compliment of the event "Over six feet tall": 7 + 5 + 8 = 20.



2.
From the diagram, we count the number of students in the intersection of the events "six foot" and "male" but not blonde: 4.

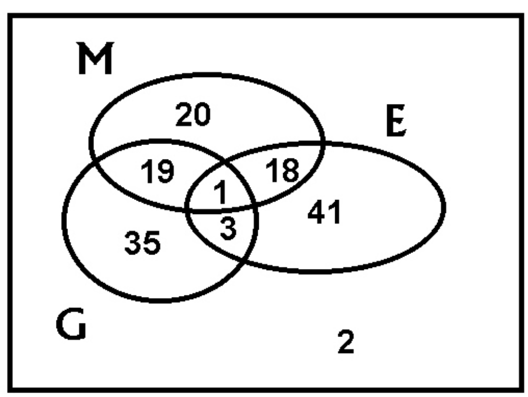
3.
From the diagram, we count the number of students that are female by counting the number of students in the compliment of the event "Male": 5 + 8 + 8 = 21.

4.
From the diagram, we count the number of female students in the intersection of the compliment of the event "six foot" and intersection of the event "blonde": 0.

5.
From the diagram, we count the number of students that are blonde along with the number of students over 6 foot: 5 + 4 + 3 + 8 + 5 = 25.

6.
From the diagram, we count the number of students in the compliment of the event "Male" and in the compliment of the event "Blonde": 5 + 8 = 13.

7.
M: The event people that use microwave ovens.
E: The event people that use electric ranges
G: The event people use gas ovens.



From the diagram, we see that 20 people only use microwave ovens.

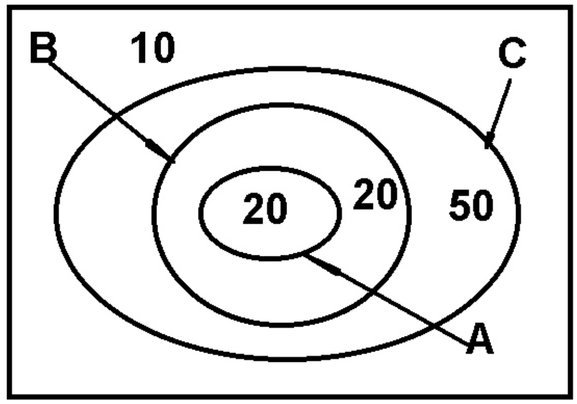
8.
From the diagram, the number of people that use

exactly one type is 20 + 35 + 41 + 2 = 98.

9.
From the diagram, the number of people that uses microwave and electric but not gas is 18.

10.
From the diagram, the number of people interviewed is

20 + 19 + 1 + 18 + 35 + 3 + 41 + 2 = 139.



11.
$\mathbf{A}'\cap\mathbf{B}$ equals the ring outside of the set **A** contained in **B.** From the diagram, $\#(\mathbf{A}'\cap\mathbf{B})$ = 20.

12.
Since $\#\mathcal{U}$ = 100 and $\#C'$ = 10, the ring outside of **B** contained in **C** must add up to 50. Therefore, from the diagram we see that #**C** = 20 + 20 + 50 = 90.

13.
$\mathbf{B}'\cap\mathbf{C}$ is the ring outside of **B** but inside of **C**. From the diagram, we see that $\#\mathbf{B}'\cap\mathbf{C} = 50$.

14.
$\#\mathbf{A}' = \#\mathcal{U} - \#\mathbf{A}$ and $\#\mathbf{B}' = \#\mathcal{U} - \#\mathbf{B}$.

Since $\#\mathbf{A} \le \#\mathbf{B}$,

$\#\mathcal{U} - \#\mathbf{B} \le \#\mathcal{U} - \#\mathbf{A}$

$\#\mathbf{B}' \le \#\mathbf{A}'$

15.
➤ a.
Step 1: Assume x is an element of $\mathbf{A}\cap\mathbf{C}'$. $\subseteq (\mathbf{A}\cap\mathbf{B}')\cup(\mathbf{B}\cap\mathbf{C}')$

Step 2: This mean x is an element of both **A** and $\mathbf{C}'$.

Step 3: Consider the following 2 cases:

Case 1: x is a member of $\mathbf{B}'$.

Since x is a member of **A**, x is a member of ($\mathbf{A}\cap\mathbf{B}'$).

Therefore, $\mathbf{A}\cap\mathbf{C}' \subseteq (\mathbf{A}\cap\mathbf{B}')\cup(\mathbf{B}\cap\mathbf{C}')$

Case 2: x is not a member of $\mathbf{B}'$.

Since x is not a member of $\mathbf{B}'$, x is a member of **B**.

Since x is a member of **B** as well as a member of $\mathbf{C}'$,

x is a member of $\mathbf{B}\cap\mathbf{C}'$.

Therefore, $\mathbf{A}\cap\mathbf{C}' \subseteq (\mathbf{A}\cap\mathbf{B}')\cup(\mathbf{B}\cap\mathbf{C}')$

➤ b.
From a, we have $\mathbf{A}\cap\mathbf{C}' \subseteq (\mathbf{A}\cap\mathbf{B}')\cup(\mathbf{B}\cap\mathbf{C}')$.

Therefore, $\#(\mathbf{A}\cap\mathbf{C}') \leq \#[(\mathbf{A}\cap\mathbf{B}')\cup(\mathbf{B}\cap\mathbf{C}')]$

Since $(\mathbf{A}\cap\mathbf{B}')\cap(\mathbf{B}\cap\mathbf{C}') = \phi$, $\#[(\mathbf{A}\cap\mathbf{B}')\cup(\mathbf{B}\cap\mathbf{C}')] = \#(\mathbf{A}\cap\mathbf{B}') + \#(\mathbf{B}\cap\mathbf{C}')$

Therefore, $\#(\mathbf{A}\cap\mathbf{C}') \leq \#(\mathbf{A}\cap\mathbf{B}') + \#(\mathbf{B}\cap\mathbf{C}')$

16.

➤a.
The set $\{\phi\}$ is a set that contains 1 element: the set ϕ**.** Therefore, the set has cardinality 1.

➤b.

The set $\{\phi, \{\phi, \{\phi,1\}\}\}$ has the following elements: ϕ, $\{\phi, \{\phi,1\}\}$. Therefore, the set has cardinality 2.