SET THEORY
Lesson 6
Venn Diagrams

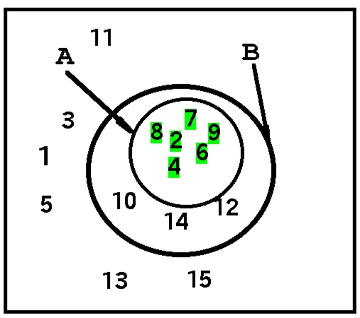
6.1 - What is a Venn Diagram?

6.1- Problem 1:

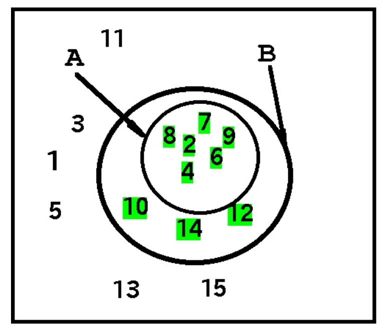
➤(a).

From the Venn diagram we see that the elements of **A** are 2,4,6,7,8,9, and **A** = {2,4,6,7,8,9}. Therefore, we shade 2,4,6,7,8,9.

(a).



(b).



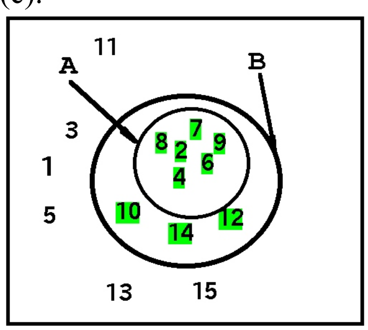
➤(b).

From the Venn diagram we see that the elements of **B** are 2,4,6,7,8,9,10,12,14, and **B** = {2,4,6,7,8,9,10,12,14}. Therefore, we shade 2,4,6,7,8,9,10,12,14.

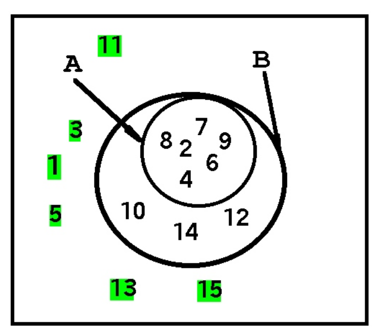
➤(c).

Because **A** ⊂ **B** we have **A**∪**B** = **B.** Therefore, we shade 2,4,6,7,8,9,10,12,14.

(c).



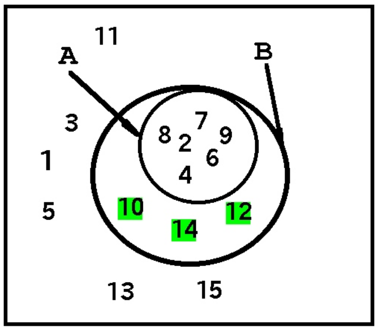
(d).



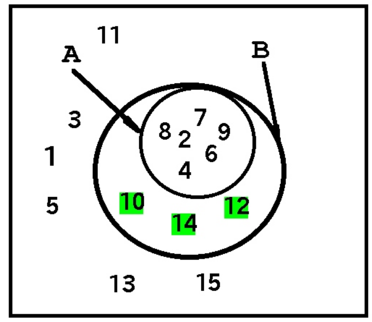
➤(d).
Because $(\mathbf{A} \cup \mathbf{B})' = \mathbf{B}'$, we shade all number outside of the set **B** : 1,3,5,11,13,15.

➤(e).
Because $\mathbf{A}' \cap \mathbf{B} = \{10,12,14\}$ we shade all numbers outside of the set **A** but in the set **B**: 10,12,14.

(e).



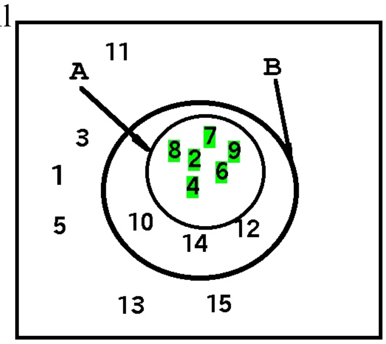
(g).



➤(f).
Because $\mathbf{A} \subset \mathbf{B}$ we have $\mathbf{A} \cap \mathbf{B}' = \phi$ .Therefore, no numbers are shaded.

➤(g).
Because $(\mathbf{A}' \cap \mathbf{B}) \cup (\mathbf{A} \cap \mathbf{B}') = (\mathbf{A}' \cap \mathbf{B}) \cup \phi = (\mathbf{A}' \cap \mathbf{B})$, we shade all numbers in the set **B** that are not in the set **A**: 10,12,14.

(h).

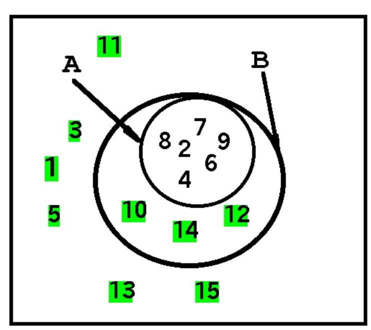


➤(h).

Solution:

Because $\mathbf{A} \cap \mathbf{B} = \mathbf{A}$ we shade the numbers in the set **A**: 2,4,6,7,8,9.

(i).



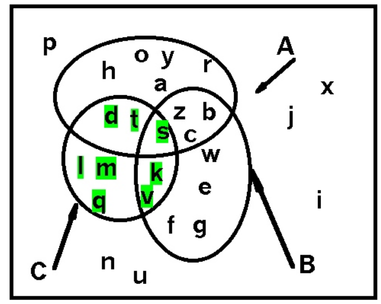
➤(i).
Because $(\mathbf{A} \cap \mathbf{B})' = \mathbf{A}'$, we shade all the numbers outside of the set **A**: 1,3,5,10,11,12,13,14,15.

6.1 - Problem 2:

➤(a).

Looking at the Venn diagram (a) we see that the set **C** = {d,t,s,k,v,l,m,q}. Therefore, we shade the letters d,t,s,k,v,l,m,q.

(a).



➤(b).

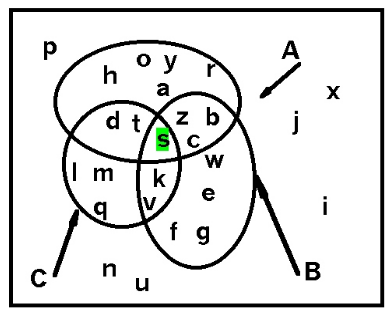
$\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}$

$=\{z,b,c,s,d,t,h,a,o,y,r\}\cap\{z,s,c,b,k,v,w,e,f,g\}\cap\{l,m.q,k,v,d,t,s\} =$

$\{s\}$.

Therefore, we shade the letter z.

(b).



➤(c).

$(\mathbf{A}\cup\mathbf{B}\cup\mathbf{C})' = [\{z,b,c,s,d,t,h,a,o,y,r\}\cup\{z,s,c,b,k,v,w,e,f,g\}\cup\{l,m.q,k,v,d,t,s\}]' =$

$\{a,b,c,d,e,f,g,h,k,l,m,o,q,r,s,t,v,w,y,z\}' = \{p,x,j,i,n,u\}$.

Therefore, we shade p,x,j,i,n,u.

➤(d).

$\mathbf{A}'\cap\mathbf{B}'\cap\mathbf{C} = (\mathbf{A}'\cap\mathbf{B}')\cap\mathbf{C} = (\mathbf{A}\cup\mathbf{B})'\cap\mathbf{C} =$

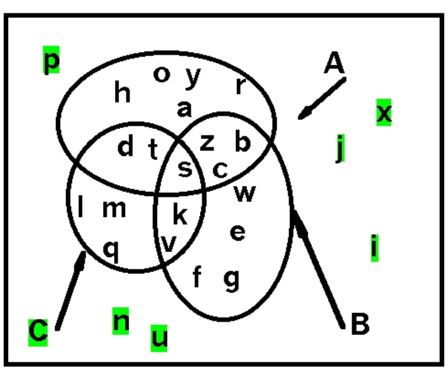
$[\{z,b,c,s,d,t,h,a,o,y,r\}\cup\{z,s,c,b,k,v,w,e,f,g\}]'\cap\{l,m.q,k,v,d,t,s\}$

$=\{z,b,c,s,d,t,h,a,o,y,r,k,v,f,g,e,w\}\}'\cap\{l,m.q,k,v,d,t,s\} =$

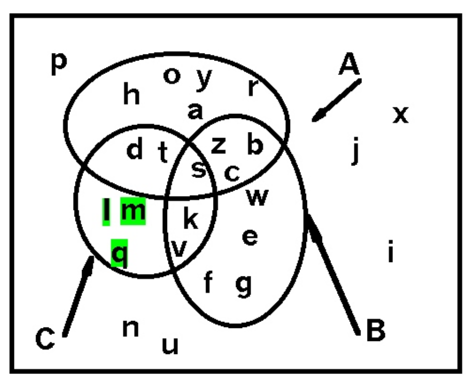
$\{l,m,q,n,u,p,x,j,i\}\cap\{l,m.q,k,v,d,t,s\} = \{l,m,q\}$.

Therefore, we shade l,m,q.

(c).

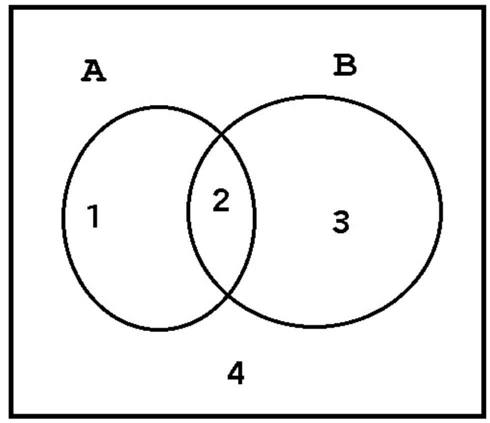


(d).



Supplementary Problems.

1.
By placing numbers in the universal set, we can list all non-empty subsets:



U = {1,2,3,4}

{1}: $\mathbf{A \cap B'}$

{2}: $\mathbf{A \cap B}$

{3}: $\mathbf{A' \cap B}$

{4}: $\mathbf{(A \cup B)'}$

{1,2}: $\mathbf{A}$

{1,3}: $\mathbf{(A \cap B') \cup (A' \cap B)}$

{1,4}: $\mathbf{(A \cap B') \cup (A \cup B)' = B'}$

{2,3}: $\mathbf{B}$

{2,4}: $\mathbf{(A \cap B) \cup (A \cup B)'}$

{3,4}: $\mathbf{(A' \cap B) \cup (A \cup B)' = A'}$

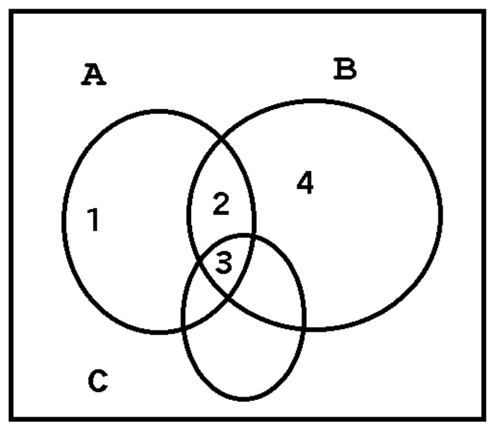
{1,2,3}: $\mathbf{A \cup B}$

{1,2,4}: $\mathbf{A \cup (A \cup B)' = B' \cup A}$

{2,3,4}: $\mathbf{B \cup (A \cup B)' = A \cup B}$

{1,3,4}: $\mathbf{(A \cap B)'}$

{1,2,3,4} = $\mathbf{A' \cup A}$



2.
{1}: $\mathbf{A \cap B'}$

{2}: $\mathbf{A \cap B \cap C'}$

{3}: $\mathbf{C}$

{4}: $\mathbf{A}'\cap\mathbf{B}$

{1,2}: $\mathbf{A}\cap\mathbf{C}'$

{1,3}: $(\mathbf{A}\cap\mathbf{B}')\cup\mathbf{C}$

{1,4}: $(\mathbf{A}\cap\mathbf{B}')\cup[\mathbf{A}'\cap\mathbf{B}]$

{2,3}: $\mathbf{A}\cap\mathbf{B}$

{2,4}: $\mathbf{B}\cap\mathbf{C}'$

{3,4}: $\mathbf{C}\cup(\mathbf{A}'\cap\mathbf{B})$

{1,2,3}: $\mathbf{A}$

{1,2,4}: $(\mathbf{A}\cup\mathbf{B})\cap\mathbf{C}'$

{1,3,4}: $[(\mathbf{A}\cap\mathbf{B}')\cup\mathbf{C}]\cup(\mathbf{A}'\cap\mathbf{B})$

{2,3,4}: $\mathbf{B}$

{1, 2, 3, 4}: $(\mathbf{A}\cup\mathbf{B}\cup\mathbf{C}$

In the following Venn diagrams, rectangles represent sets **A**,**B**,**C**. Using **A**,**B**,**C**, unions, intersections, and compliments represent the shaded areas.

3.
The top left shaded area is $(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')$.

The bottom right shaded area is $(\mathbf{A}\cap\mathbf{C}\cap\mathbf{B}')$.

Therefore, the total shaded area is

$(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{A}\cap\mathbf{C} \cap \mathbf{B}')$.

4.
The top right shaded area is $(\mathbf{A}\cap\mathbf{B} \cap \mathbf{C}')$.

The bottom shaded area is $(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')$

The bottom right shaded area is $(\mathbf{B}\cap\mathbf{C}\cap\mathbf{A}')$

Therefore, the total area is

$(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{A}\cap\mathbf{C}\cap\mathbf{B}')\cup(\mathbf{B}\cap\mathbf{C}\cap\mathbf{A}')$.

5.
The left shaded area is $\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C})'$.

The right top shaded area is $\mathbf{B}\cap(\mathbf{A}\cup\mathbf{C})'$.

The bottom right shaded area is $\mathbf{C}\cap(\mathbf{A}\cup\mathbf{B})'$.

Therefore, the total shaded area is $[\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C})']\cup[\mathbf{B}\cap(\mathbf{A}\cup\mathbf{C})']\cup[\mathbf{C}\cap(\mathbf{A}\cup\mathbf{B})']$.

6.
$(\mathbf{A}\cap\mathbf{B}) = \{3, 4\}$

$(\mathbf{A}\cap\mathbf{B})\cap\mathbf{C} = \{3, 4\}\cap\{5, 7, 4, 6\} = \{4\}$

$(\mathbf{B}\cap\mathbf{C}) = \{4, 6\}$

$\mathbf{A}\cap(\mathbf{B}\cap\mathbf{C}) = \{1, 3, 4, 5\}\cap\{4, 6\} = \{4\}$

Therefore, $(\mathbf{A}\cap\mathbf{B})\cap\mathbf{C} = \mathbf{A}\cap(\mathbf{B}\cap\mathbf{C})$.

7.
$(\mathbf{A}\cup\mathbf{B}) = \{1,3,4,5,2,6\}$

$(\mathbf{A}\cup\mathbf{B})\cup\mathbf{C} = \{1,3,4,5,2,6\}\cup\{4, 5, 6, 7\} = \{1,3,4,5,2,6, 7\}$

$(\mathbf{B}\cup\mathbf{C}) = \{2, 3, 4, 6, 5, 7\}$

$\mathbf{A}\cup(\mathbf{B}\cup\mathbf{C}) = \{1, 3, 4, 5\}\cup\{2, 3, 4, 6, 5, 7\} = \{1, 3, 4, 5, 2, 6, 7\}$

Therefore, $(\mathbf{A}\cup\mathbf{B})\cup\mathbf{C} = \mathbf{A}\cup(\mathbf{B}\cup\mathbf{C})$.

8.
$\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C}) = \{1, 3, 4, 5\}\cap\{2, 3, 4, 6, 5, 7\} = \{3, 4, 5\}$

$(\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{C}) = \{3, 4\}\cup\{4, 5\} = \{3, 4, 5\}$

Therefore, $\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C}) = (\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{C})$

9.
$\mathbf{A}\cup(\mathbf{B}\cap\mathbf{C}) = \{1, 3, 4, 5\}\cup\{4, 6\} = \{1, 3, 4, 5, 6\}$

$(\mathbf{A}\cup\mathbf{B})\cap(\mathbf{A}\cup\mathbf{C}) = \{1, 3, 4, 5, 2, 6\}\cap\{1, 3, 4, 5, 6, 7\} = \{1, 3, 4, 5, 6\}$

Therefore, $\mathbf{A}\cup(\mathbf{B}\cap\mathbf{C}) = (\mathbf{A}\cup\mathbf{B})\cap(\mathbf{A}\cup\mathbf{C})$.

10.
$(\mathbf{A}\cup\mathbf{B})' = \{1,3,4,5,2,6\}' = \{7, 8\}$

$\mathbf{A}'\cap\mathbf{B}' = \{1, 3, 4, 5\}'\cap\{2, 3, 4, 6\}' = 2, 6, 7, 8\}\cap\{1, 5, 7, 8\} = \{7, 8\}$

Therefore, $(\mathbf{A}\cup\mathbf{B})' = \mathbf{A}'\cap\mathbf{B}'$**.**

11.
$(\mathbf{A}\cap\mathbf{B})' = \{3, 4\}' = \{1, 2, 5, 6, 7, 8\}$

$\mathbf{A}'\cup\mathbf{B}' = \{2, 6, 7, 8\}\cup\{1, 5, 7, 8\} = \{1, 2, 5, 6, 7, 8\}$.

12.
Using the above venn diagram, the following list is the subsets corresponding to the numbers.

1: $\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C})' = \mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}'$

2: $\mathbf{B}\cap(\mathbf{A}\cup\mathbf{C})' = \mathbf{A}'\cap\mathbf{B}\cap\mathbf{C}'$

7: $\mathbf{C}\cap(\mathbf{A}\cup\mathbf{B})' = \mathbf{A}'\cap\mathbf{B}'\cap\mathbf{C}$

3: $\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}'$

6: $\mathbf{A}'\cap\mathbf{B}\cap\mathbf{C}$

5: $\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}$

4: $\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}$

$\mathbf{A}\cup\mathbf{B}\cup\mathbf{C} = (\mathbf{A}\cap\mathbf{B}\cap\mathbf{C})\cup[(\mathbf{A}'\cap\mathbf{B}\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')]\cup$

$[(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}')\cup(\mathbf{A}'\cap\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{A}'\cap\mathbf{B}'\cap\mathbf{C})]$