SET THEORY
Lesson 5
Set Operations

5.1 - What is the Union of Two or More Sets?

5.1 - Problem 1:
For the union, we need to combine together all the elements from sets **G** and **D:**

Step 1: **G**∪**D =** {airplanes,automobles,skates}∪{horses,skates,tricycles,rockets}

Step 2: Combining these two sets together we remove one of the duplicated element Skates:

G∪**D =** {airplanes,automobles,skates,horses,tricycles,rockets}}.

5.1 - Problem 2:
For the union, we need to combine together all the elements from sets **F** and **D:**

Step 1: **F**∪**D =** {Califonia,Washington,a,b,Howard}∪{1,2,3,4,5}

Step 2: Combining these distinct elements together:

F∪**D =** {Califonia,Washington,a,b,Howard,1,2,3,4,5}

5.1 - Problem 3:
For the union, we need to combine together all the elements from sets **K**, **D**, and **R**

Step 1 **K**∪**D**∪**R** = {a,b,c}∪{c,d,e}∪{e,f,g}

Step 2: Combine all elements after eliminating the duplications: c,e.

K∪**D**∪**R** = {a,b,c}∪{c,d,e}∪{e,f,g} = {a,b,c,d,e,f,g}

5.1 - Problem 4:
Step 1: **T**∪**P**∪**T = T**∪**T**∪**P**

Step 2:**T**∪**T = T**

Step 3: **T**∪**P**∪**T = T**∪**T**∪**P** = **T**∪**P**

Step 4: **T**∪**P**∪**T** = **T**∪**P** = {(a,b,c,d) }∪{(a,b,d,c),(a,c,b,d),(a,d,c,b)}

Step 5: Combine each of these elements. Note there are no duplications.

$\mathbf{T}\cup\mathbf{P}\cup\mathbf{T} = \mathbf{T}\cup\mathbf{P} =$ {(a,b,c,d),(a,b,d,c),(a,c,b,d),(a,d,c,b)}.

5.1 - Problem 5:
Step 1: $\mathbf{W}\cup\mathbf{X} =$ {(1,1),(1,2),(2,1),(2,2)}∪ {(1,1),(2,2)}

Step 2: There are two duplications (1,1), (2,2) which have to be eliminated:

$\mathbf{W}\cup\mathbf{X} =$ {(1,1),(1,2),(2,1),(2,2)}

5.1 - Problem 1:
Since the empty set ϕ has no elements, $\mathbf{E} = \{1,2,3,4,...\}\cup\phi = \{1,2,3,4,...\}$.

5.2 - What is the Intersection of Two or More Sets?

5.2 - Problem 1:
Step 1: Find, those elements that are in both sets **A** and **B**: Miami, New York.

Step 2: Since $\mathbf{A}\cap\mathbf{B}$ is the set made up of elements that are in both **A** and **B**: Miami, New York

Step 3: $\mathbf{A}\cap\mathbf{B} =$ {Miami,New York}.

5.2 - Problem 2:

Step 1: $\mathbf{B}\cap\mathbf{A}\cap\mathbf{B} = \mathbf{B}\cap\mathbf{B}\cap\mathbf{A}$

Step 2: $\mathbf{B}\cap\mathbf{B} = \mathbf{B}$

Step 3: $\mathbf{B}\cap\mathbf{A}\cap\mathbf{B} = (\mathbf{B}\cap\mathbf{B})\cap\mathbf{A} = \mathbf{B}\cap\mathbf{A}$

Step 2: Find, those elements that are in both sets **A** and **B**: (1,2,3),(1,3,5).

Step 3: Since $\mathbf{A} \cap \mathbf{B}$ is the set made up of elements that are in both **A** and **B**:

$\mathbf{B}\cap\mathbf{A} =$ {(1,2,3),(1,3,5)}.

Step 4: $\mathbf{B} \cap\mathbf{A}\cap\mathbf{B} = \mathbf{B}\cap\mathbf{A} =$ {(1,2,3),(1,3,5)}.

5.2 - Problem 3:
Step 1: Find, those elements that are in both sets **E** and **D**: Miami, New York.

Step 2: Since $\mathbf{E}\cap\mathbf{D}$ is the set made up of elements that are in both **E** and **D**:{1,2,3}.

Step 3: **E**∩**D =** {{1,2,3}}.

5.2 - Problem 4:

Step 1: Find, those elements that are in both sets **R** and **E**. There are no common elements. (Note: The element Mary Smith in **R** and the element Mary in **E** are different elements.)

Step 2: Since **R** ∩ **E** is the set made up of elements that are in both **R** and **E**, **R**∩**E** = ϕ.

5.2 - Problem 5:
Step 1: Find, those elements that are in both sets **R**, **I**,**M**: {4,5,6,...}.

Step 2: Since **R**∩**M**∩**I** is the set made up of elements that are in both **R** , **M**, and **I**:{4,5,6,..}.

Step 3: **R**∩**M**∩**I =** {4,5,6,...}

5.2 - Problem 6:
Step 1: Find, those elements that are in both sets **T** and **S**: (1,1).

Step 2: Since **E**∩**D** is the set made up of elements that are in both **E** and **D**:(1,1).

Step 3: **S**∩**T** = {(1,1)}

5.3 - What is a Subset?

5.3 - Problem 1:
The set **F** has the element e which is not a element of the set **B.** Therefore, the set **F** is not a subset of **B.**

5.3 - Problem 2:
The elements of this set are {a,b} and {a,c}.

Step 1: Since the empty set ϕ is a subset of all sets, ϕ is a subset of **U.**

Step 2: Sets made up of one element of **U**: {{a,b}}, {{a,c}}

Step 3: Set made up of two elements of **U**: {{a,b},{a,c}} (note: a set is a subset of itself.)

Step 4: Therefore, the following are all subsets of **U**, ϕ, {{a,b}},{{a,c}}, {{a,b},{a,c}}

5.3 - Problem 3:
The elements of this set are 1,2,3.

Step 1: Since the empty set ϕ is a subset of all sets, ϕ is a subset of **U.**

Step 2: Sets made up of one element of **U**: {1},{2},{3}.

Step 3: Set made up of two elements of **U**: {1,2},{1,3},{2,3}.

Step 4: Set made up of three elements of **U**: {1,2,3}.

Step 5: Therefore, all subsets of **U** are ϕ {1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}.

5.3 - Problem 4:
The elements of this set are (r,r,b),(r,b,r),(b,r,r),(r,r,r).

Step 1: Since the empty set ϕ is a subset of all sets, ϕ is a subset of **E.**

Step 2: Sets made up of one element of **E**: {(r,r,b)},{(r,b,r)},{(b,r,r)},{(r,r,r)}.

Step 3: Set made up of two elements of **E**: {(b,r,r),(r,b,r)},{(b,r,r),(r,r,b)},{(r,r,b),(r,b,r)},

{(r,r,r),(r,b,r)},{(r,r,r),(r,r,b)},{(r,r,r),(b,r,r)}

Step 4: Sets made up of 3 elements of **E**:

{(r,r,b),(r,b,r),(b,r,r)},{(b,r,r),(r,b,r),(r,r,r)},

{(b,r,r),(r,r,b),(r,r,r)},{(r,r,b),(r,b,r),(r,r,r)},

Step 5: Set made up of four elements of **E**:

{(r,r,b)},{(r,b,r)},{(b,r,r)},{(r,r,r)}

Step 6: Therefore, all subsets of **E** are ϕ, {(r,r,b)},{(r,b,r)},{(b,r,r)},{(r,r,r)},

{(b,r,r),(r,b,r)},{(b,r,r),(r,r,b)},{(r,r,b),(r,b,r)},

{(r,r,r),(r,b,r)},{(r,r,r),(r,r,b)},{(r,r,r),(b,r,r)},

{(r,r,b),(r,b,r),(b,r,r)},{(b,r,r),(r,b,r),(r,r,r)},

{(b,r,r),(r,r,b),(r,r,r)},{(r,r,b),(r,b,r),(r,r,r)},

{(r,r,b),(r,b,r),(b,r,r)},{(r,r,b)},{(r,b,r)},{(b,r,r)},{(r,r,r)}

5.4 - What is a Proper Subset?

5.4 - Problem 1:
Since 8 is an element of the set **F** but it is not an element of the set **B**, **F** is not a subset of **B.**

5.4 - Problem 2:
$\phi \subset$ {Billy,Sally,Jill}
{Billy} $\subset$ {Billy,Sally,Jill}
{Sally} $\subset$ {Billy,Sally,Jill}
{Jill} $\subset$ {Billy,Sally,Jill}
{Billy,Sally} $\subset$ {Billy,Sally,Jill}
{Billy,Jill} $\subset$ {Billy,Sally,Jill}
{Sally,Jill} $\subset$ {Billy,Sally,Jill}
Therefore, the following are the proper subsets of **W:**
ϕ, {Billy},{Sally},{Jill},{Billy,Sally},{Billy,Jill},{Sally,Jill}.

Problem 5.3:
$\phi \subset$ {(1,1,1),(2,2,2),(1,2,1)}
{(1,1,1)} $\subset$ {(1,1,1),(2,2,2),(1,2,1)}
{(2,2,2)} $\subset$ {(1,1,1),(2,2,2),(1,2,1)}
{(1,2,1)} $\subset$ {(1,1,1),(2,2,2),(1,2,1)}
{(1,1,1),(2,2,2)} $\subset$ {(1,1,1),(2,2,2),(1,2,1)}
{(1,1,1),(1,2,1)} $\subset$ {(1,1,1),(2,2,2),(1,2,1)}
{(2,2,2),(1,2,1)} $\subset$ {(1,1,1),(2,2,2),(1,2,1)}

5.5 - What is a Universal Set?

5.5 - Problem 1:
Since:
{horses} $\subseteq$ {horses,cattle,pigs}

{cattle,pigs} $\subseteq$ {horses,cattle,pigs}

The set $\mathcal{U}$ = {horses,cattle,pigs} is a universal set.

5.5 - Problem 2:
The set **B** has the element cats that is not an element of the set {horses,cattle,pigs}. Therefore, **B** is not a subset of {horses,cattle,pigs} and the set {horses,cattle,pigs} is not a universal set.

5.5 - Problem 3:
The easiest way to construct the universal set $\mathcal{U}$ is to take the union of the sets **Q** and **P** :
Q∪P = {(h,h),(h,t),(t,h),(t,t),h,t}.

Since **Q** ⊆ **Q∪P** and **P** ⊆ **Q∪P** , it follows that $\mathcal{U}$ **= Q∪P** = {(h,h),(h,t),(t,h),(t,t),h,t} is the smallest universal set containing the sets **P** and **Q**.

5.5 - Problem 4:
The easiest way to construct the universal set $\mathcal{U}$ is to take the union of the sets **Q** , **P** and **E**:
Q∪P∪E = {(h,h),(h,t),(t,h),(t,t)}.

Since **Q** ⊆ **Q∪P∪E** , **P** ⊆ **Q∪P∪E** and **E** ⊆ **Q∪P∪E**, it follows that

$\mathcal{U}$ **= Q∪P∪E** = {(h,h),(h,t),(t,h),(t,t)} is the smallest universal set containing the sets **P, Q** and **E**. (Note: **Q** = $\mathcal{U}$)

5.6 - What is the Compliment of a Set?

5.6 - Problem 1:
Step 1: **D** = {a,d} ⊆ {a,b,c,d}

Step 2: **D'** is the set of elements that are in the universal set {a,b,c,d} but not in **D** = {a,d}: {b,c}.

Step 3: Therefore, **D'** = {b,c}.

5.6 - Problem 2:
Step 1: **A** = {(h,h,t),(h,t,h),(t,h,h)} ⊆ {(h,h,h),(t,t,t),(h,h,t),(h,t,h),(t,h,h),(t,t,h),(t,h,t),(h,t,t)}

Step 2: A' is the set of elements that are in the universal set

{(h,h,h),(t,t,t),(h,h,t),(h,t,h),(t,h,h),(t,t,h),(t,h,t),(h,t,t)} but not in

A = {(h,h,t),(h,t,h),(t,h,h)}:{(h,h,h),(t,t,t),(t,t,h),(t,h,t),(h,t,t)}

Step 3: Therefore,

A' = {(h,h,h),(t,t,t),(t,t,h),(t,h,t),(h,t,t)}

5.7 - Combining operators.

5.7 - Problems 1:
Step 1: **D**∪**B** = {1,2,3,4,5} ∪ {1,3,5,7,9,**...**,99} = {1,2,3,4,5,7,9,11,...,99}.

Step 2: (**D**∪**B**)' = {1,2,3,4,5,7,9,11,...,99}' = {6,8,10,12,...,100}.

Step 3: Therefore, (**D**∪**B**)' = {6,8,10,12,...,100}.

5.7 - Problem 7.2:
Step 1: **D**∩**B** = {1,2,3,4,5}∩{1,3,5,7,9,**...**,99} = {1,3,5}.

Step 2: (**D**∩**B**)' = {1,3,5}' = {2,4,6,7,8,9,10,11,...,100}.

Step 3: Therefore, (**D**∩**B**)' = {2,4,6,7,8,9,10,11,...,100}.

5.7 - Problem 3:
Step 1:

A∪**B =** {2,4,6,8,10,...100} ∪ {1,3,5,7,9,**...**,99} = {1,2,3,4,...,100}.

Step 2:(**A**∪**B**)∩**D** = {1,2,3,4,...,100}∩{1,2,3,4,5} = {1,2,3,4,5}.

Step 3: Therefore, (**A**∪**B**)∩**D** = {1,2,3,4,5}.

5.7 - Problem 4:
Step 1: **A**∩**B =** {2,4,6,8,10,...100}∩{1,3,5,7,9,...,99} = ϕ

Step 2:(**A**∩**B**)∪**D** = ϕ∪{1,2,3,4,5} = {1,2,3,4,5}.

Step 3: Therefore, (**A**∩**B**)∪**D** = ϕ∪{1,2,3,4,5} = {1,2,3,4,5}.

5.7 - Problem 5:
Step 1: **A**∪**D =** {2,4,6,8,10,...100}∪{1,2,3,4,5} = {1,2,3,4,5,6,8,10,12,...,100}

Step 2: (**A**∪**D**)**' =** {1,2,3,4,5,6,8,10,12,...,100}' = {7,9,11,13,15,...,99}

Step 3: (**A**∪**D**)'∩**B =** {7,9,11,13,15,...,99}∩{1,3,5,7,9,11,...,99} = {7,9,11,...99}

Step 4: Therefore, (**A**∪**D**)'∩**B** = {7,9,11,...99}.

5.7 - Problem 6:
Step 1: **A**∪**B** = {2,4,6,8,10,...100}∪{1,3,5,...,99} = {1,2,3,...99,100}

Step 2: **D'** = {6,7,8,9,10,11...100}

Step 3:(**A**∪**B**)∩**D'** = {1,2,3,...,99,100}∩{6,7,8,9,10,11,...,100} = {6,7,8,9,10,11,...,100}

Step 4: Therefore,(**A**∪**B**)∩**D'**= {6,7,8,9,10,11,...,100}

5.8 - DeMorgan Laws

5.8 - Problem 1:
➤(a).
Left Side:

Step 1: **A**∪**B** = {(h,h),(h,t),(t,h)}∪{(h,t),(t,h)} = {(h,h),(h,t),(t,h)}

Step 2: (**A**∪**B**)' = {(h,h),(h,t),(t,h)}}' = {(t,t)}

Right Side:

Step 1: **A'**∩**B'** = {(t,t)}∩{(t,t),(h,h)} = {(t,t)}.

Step 2: Therefore, (**A**∪**B**)' = **A'**∩**B'**.

➤(b).
Left Side:

Step 1: **A**∩**B** = {(h,h),(h,t),(t,h)}∩{(h,t),(t,h)} = {(h,t),(t,h)}

Step 2: (**A**∩**B**)' = {(h,t),(t,h)}' = {(t,t),(h,h)}

Right Side:

Step 1: **A'**∪**B' =** {(t,t),(h,h)}.

Step 2: Therefore, (**A**∩**B**)' = **A'**∪**B'**.

Supplementary Problems.

1.
One way of showing these two sets are equal is to assume they are not equal and show a logical error. If these two sets are not equal then either **A** contains a element that is not in **B** or **B** contains an element that is not in **A.** Assume **A** contains an element that is not in **B**. This would mean that **A** cannot be a subset of **B.** But we know that **A** is a subset of **B.** This is a logical error.

Therefore, **A** = **B.**

2.
We use the result in problem 1: If **A** ⊆ **B** and **B** ⊆ **A** then **A** = **B.**

Step 1: **D**∩**D** ⊆ **D**

Step 2: **D** ⊆ **D** ∩**D**

Step 3: Therefore, **D**∩**D** = **D.**

3.
Assume x is an element that lies in the set **F**∩**F**′. This means that x lies in both **F** and **F'**. But this is impossible since an element cannot lie both in a set and at the same time outside the set. Therefore, **F**∩**F**′ = ϕ

4.
Step 1: Assume x is a arbitrary element of the universal set 𝔘.

Step 2: **F**∪**F**′ ⊆ 𝔘

Step 3: Now if x does not lie in **F** then it must lie in **F'.** Therefore, 𝔘 ⊆ **F**∪**F**′ .

Step 4: Since **F**∪**F**′ ⊆ 𝔘 and 𝔘 ⊆ **F**∪**F**′ we conclude **F**∪**F**′ = 𝔘.

5.
Step 1: ϕ′ ⊆ 𝔘

Step 2: Assume x is an arbitrary element of 𝔘 **.** Since x cannot be an element of ϕ, we conclude that x is an element of ϕ ′ Therefore, 𝔘 ⊆ ϕ′ .

Step 3: Therefore, ϕ′ = 𝔘. .

6.
Step 1: ϕ∩**G** ⊆ ϕ

Step 2: Since ϕ∩**G** cannot be a proper subset of ϕ, we conclude that ϕ∩**G** = ϕ

7.
Step 1: ϕ∪**G** ⊆ **G**

Step 2: $\mathbf{G} \subseteq \phi \cup \mathbf{G}$

Step 3: $\phi \cup \mathbf{G} = \mathbf{G}$

8.
Step 1: $\mathfrak{U} \cup \mathbf{G} \subseteq \mathfrak{U}$

Step 2: $\mathfrak{U} \subseteq \mathfrak{U} \cup \mathbf{G}$

Step 3: Therefore, $\mathfrak{U} \cup \mathbf{G} = \mathfrak{U}$.

9.
Step 1: $\mathfrak{U} \cap \mathbf{G} \subseteq \mathbf{G}$

Step 2: $\mathbf{G} \subseteq \mathfrak{U} \cap \mathbf{G}$

Step 3: Therefore, $\mathfrak{U} \cap \mathbf{G} = \mathbf{G}$.

10.
The compliment of the universal set is the set of elements that are not in the universal set. But all elements are in the universal set. Therefore, $\mathfrak{U}' = \phi$.

11.
Step 1: Let **A = F'.** Then **A** are all elements of the universal set outside of **F.**

Step 2: The set **A'** are all elements of the universal set that are outside of **A** and therefore in **F.**

Step 3: Since **A' = F''** **,** we conclude that **F'' = F.**

12.
$\mathbf{A}_0 = \{-100, -99, ..., -1, 0, 1, 2, ..., 100\}$

$\mathbf{A}_1 = \{-99, -98 ..., -1, 0, 1, 2, ...,98,99\}$

$\mathbf{A}_2 = \{-98, -97 ..., -1, 0, 1, 2, ...,97,98\}$

::
$\mathbf{A}_{99} = \{-1, 0, 1\}$

$\mathbf{A}_{100} = \{0\}$

Since $\mathbf{A}_1 \supset \mathbf{A}_2 \supset \mathbf{A}_3 \supset \ldots \supset \mathbf{A}_{99} \supset \mathbf{A}_{100}$,the set $\mathbf{A}_{100} = \{0\}$ is in each of the sets.

Therefore, $\mathbf{A}_0 \cap \mathbf{A}_1 \cap \ldots \cap \mathbf{A}_{100} = \{0\}$.

13.

A is a subset of **B** if all the elements of **A** are in **B**.

From this definition the following holds:

If **A** is not a subset of **B** then **A** must, by definition, have at least 1 element not in **B**.

There is no restriction on the sets **A** and **B** for this definition. And the following proof only follows from this definition of a subset:

Step 1: Assume **A** is an arbitrary set.

Step 2: Since ϕ is also a set, there are only 2 possibilities:

1. ϕ is a subset of **A** or

2. ϕ is not a subset of **A**.

If 1. is true, we are done.

If 1. is not true, then 2. must be true.

If 2. is true then, since ϕ is a set and from the definition of subsets, ϕ must have an element in it that is not in **A**.

But by definition, ϕ has no elements.

Therefore 2. can not be true. And if 2. is not true then 1. must be true.

14.

➤ a.

Step 1: By DeMorgan: $(\mathbf{A} \cup \mathbf{B})' \cap \mathbf{B} = (\mathbf{A}' \cap \mathbf{B}') \cap \mathbf{B}$

Step 2: From the definition of $\cap$: $\mathbf{A}' \cap (\mathbf{B}' \cap \mathbf{B}) = \mathbf{A}' \cap \phi = \phi$

➤ b.

Step 1: By DeMorgan: $(\mathbf{A} \cap \mathbf{B})' \cup \mathbf{B} = (\mathbf{A}' \cup \mathbf{B}') \cup \mathbf{B}$

Step 2: From the definition of $\cup$:, $\mathbf{A}' \cup (\mathbf{B}' \cup \mathbf{B}) = \mathbf{A}' \cup \mathfrak{U} = \mathfrak{U}$.

15.

$\mathbf{E} = (\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) \cup (\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}') \cup$
$(\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}') \cup (\mathbf{A}' \cap \mathbf{B} \cap \mathbf{C}) \cup (\mathbf{A}' \cap \mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{A}' \cap \mathbf{B} \cap \mathbf{C}') \cup (\mathbf{A}' \cap \mathbf{B}' \cap \mathbf{C}')$

$\mathbf{A}' = \{0,1,5,6,7,10\}$
$\mathbf{B}' = \{0,2,4,6,8,10\}$

$\mathbf{C}' = \{0,2,4,5,7,8,9\}$

$(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) = \{3\}$

$(\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}) = \phi$

$(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}') = \{9\}$

$(\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}') = \{2,4,8\}$

$(\mathbf{A}' \cap \mathbf{B} \cap \mathbf{C}) = \{1\}$

$(\mathbf{A}' \cap \mathbf{B}' \cap \mathbf{C}) = \{6,10\}$

$(\mathbf{A}' \cap \mathbf{B} \cap \mathbf{C}') = \{5,7\}$

$(\mathbf{A}' \cap \mathbf{B}' \cap \mathbf{C}') = \{0\}$

$\mathbf{E} = (\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) \cup (\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}') \cup$
$(\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}') \cup (\mathbf{A}' \cap \mathbf{B} \cap \mathbf{C}) \cup (\mathbf{A}' \cap \mathbf{B}' \cap \mathbf{C}) \cup (\mathbf{A}' \cap \mathbf{B} \cap \mathbf{C}') \cup (\mathbf{A}' \cap \mathbf{B}' \cap \mathbf{C}') = \mathfrak{U}$

16.
.
To show these are true, we use the notation a $\in$ **A** means a is an element of the set **A**.

➤ **a**.
Step 1: a $\in$ **A** . Then a is not in $\mathbf{A}'$. Therefore, a $\in \mathbf{A}''$ Conclusion $\mathbf{A} \subseteq \mathbf{A}''$

Step 2: a $\in \mathbf{A}''$. Therefore, a is not in $\mathbf{A}'$ and therefore, a $\in$ **A.** Conclusion $\mathbf{A}'' \subseteq \mathbf{A}$.

Therefore, $\mathbf{A} = \mathbf{A}''$

➤ **b**.
Step 1: Assume a $\in (\mathbf{A} \cup \mathbf{B})'$

The following are 2 possibilities: a $\in$ **A** or a $\in \mathbf{A}'$

If a $\in$ **A** then a $\in \mathbf{A} \cup \mathbf{B}$. But this contradicts a $\in (\mathbf{A} \cup \mathbf{B})'$. Therefore, a $\in \mathbf{A}'$.

The following are 2 possibilities: a ∈ **B** or a ∈ **B**'

If a ∈ **B** then a ∈ **A**∪**B**. But this contradicts a ∈ (**A**∪**B**)' . Therefore, a ∈ **B**'.

a ∈ **A** '∩ **B**' . Therefore, (**A**∪**B**)' ⊆ **A** '∩ **B**' .

Step 2: Assume a ∈ **A** '∩**B**'. Therefore, a ∈ **A**′ and a ∈ **B**′**.**

There are 2 possibilities: a ∈ (**A**∪**B**) or a ∈ (**A**∪**B**)′

Assume a ∈ (**A**∪**B**). This means that a ∈ **A** or a ∈ **B**.

We see this is not possible since a ∈ **A**′ and a ∈ **B**′**.**

Therefore, a ∈ (**A**∪**B**)′

We conclude **A** '∩ **B**' ⊆ (**A**∪**B**)'

Since **A** '∩ **B**' and (**A**∪**B**)' are subsets of each other we conclude **A** '∩ **B**' = (**A**∪**B**)'

➤**c**.
Step 1: Assume a ∈ (**A**∩**B**)'.

If a ∈ **A** then a ∈ **B**'. There a ∈ **A**'∪**B**'

If a ∈ **B** then a ∈ **A**'. There a ∈ **A**'∪**B**'

(**A**∩**B**)' ⊆ **A**'∪**B**'

Step 2: Assume a ∈ **A**'∪**B**'

a ∈ **A**' or a ∈ **B**'

a ∈ (**A**∩**B**)' or a ∈ (**A**∩**B**).

If a ∈ (**A**∩**B**) then a ∈ **A** and a ∈ **B.**

This is a contradiction. Therefore, a ∈ (**A**∩**B**)'

A'∪**B**' ⊆ (**A**∩**B**)'

distributive laws:

➤**d**.
Step 1: a ∈ **A**∩(**B**∪**C**)

a ∈ **A** and a ∈ (**B**∪**C**)

There are 2 possibilities: a ∈ (**A**∩**B**) or a ∈ (**A**∩**C**)

Therefore, a ∈ (**A**∩**B**)∪(**A**∩**C**) and we conclude **A**∩(**B**∪**C**) ⊆ (**A**∩**B**)∪(**A**∩**C**).

Step 2: a ∈ (**A**∩**B**)∪(**A**∩**C**)

a ∈ **A**

There are 2 possibilities: a ∈ (**A**∩**B**) or a ∈ (**A**∩**C**)

If a ∈ (**A**∩**B**) then a ∈ **B** and it follows a ∈ (**B** ∪**C**).

Therefore, a ∈ **A**∩(**B**∪**C**)

If a ∈ (**A**∩**C**) then a ∈ **C** and it follows a ∈ (**B** ∪**C**)

Again, a ∈ **A**∩(**B**∪**C**).

Therefore, (**A**∩**B**)∪(**A**∩**C**) ⊆ **A**∩(**B**∪**C**)

➤**e**.
[**A**∪(**B**∩**C**)]′ = **A**′ ∩(**B**′∪**C**′) = (**A**′ ∩**B**′)∪(**A**′ ∩ **C**′)

[(**A**∪**B**)∩(**A**∪**C**)]′ = (**A**′∩**B**)′∪(**A**′∩**C**′)

Since [**A**∪(**B**∩**C**)]′ = [(**A**∪**B**)∩(**A**∪**C**)]′, it follows that **A**∪(**B**∩**C**) = (**A**∪**B**)∩(**A**∪**C**)

➤**f**.
From the distributive law,

(**A**∪**B**)∩(**C**∪**D**) = [(**A**∪**B**)∩**C**]∪[(**A**∪**B**)∩**D**] =
(**A**∩**C**)∪(**B**∩**C**)∪(**A**∩**D**)∪(**B**∩**D**)