Descriptive Statistics
Lesson 3
Measuring Data Variation

3.1 - What is Data Variation?

3.1 - Problem 1:

➤(a).

The range is the difference between the largest and smallest value. From the data we see that the number six is the largest number and the number one is the smallest. There the range is 6 - 1 = 5.

➤(b).

Step 1: Compute the average value of this sequence of numbers:

$$\overline{X} = \frac{4+5+6+4+2+4+2+2+4+4+2+4+1+1+3+4+5+3+3+4}{20} = \frac{67}{20} = 3.35$$

To compute the absolute mean variation we complete the following table:

Data X	$\lvert X - \bar{x} \rvert$
4	$\lvert 4 - 3.35 \rvert = 0.65$
5	$\lvert 5 - 3.35 \rvert = 1.65$
6	$\lvert 6 - 3.35 \rvert = 2.65$
4	$\lvert 4 - 3.35 \rvert = 0.65$
2	$\lvert 2 - 3.35 \rvert = 1.35$
4	$\lvert 4 - 3.35 \rvert = 0.65$
2	$\lvert 2 - 3.35 \rvert = 1.35$
2	$\lvert 2 - 3.35 \rvert = 1.35$
4	$\lvert 4 - 3.35 \rvert = 0.65$
4	$\lvert 4 - 3.35 \rvert = 0.65$
2	$\lvert 2 - 3.35 \rvert = 1.35$
4	$\lvert 4 - 3.35 \rvert = 0.65$
1	$\lvert 1 - 3.35 \rvert = 2.35$
1	$\lvert 1 - 3.35 \rvert = 2.35$
3	$\lvert 3 - 3.35 \rvert = 0.35$
4	$\lvert 4 - 3.35 \rvert = 0.65$
5	$\lvert 5 - 3.35 \rvert = 1.65$
3	$\lvert 3 - 3.35 \rvert = 0.35$
3	$\lvert 3 - 3.35 \rvert = 0.35$
4	$\lvert 4 - 3.35 \rvert = 0.65$
$\bar{x} = 3.35$	**AMV** = 22.3/20 ≈ 1.12

➤(c).
To compute the standard deviation, complete the following table:

Step 1:

Data X	$(X - \bar{x})^2$
4	$(4 - 3.35)^2 = 0.65^2 = 0.4225$
5	$(5 - 3.35)^2 = 1.65^2 = 2.7225$
6	$(6 - 3.35)^2 = 2.65^2 = 7.0225$
4	$(4 - 3.35)^2 = 0.65^2 = 0.4225$
2	$(2 - 3.35)^2 = 1.35^2 = 1.8225$
4	$(4 - 3.35)^2 = 0.65^2 = 0.4225$
2	$(2 - 3.35)^2 = 1.35^2 = 1.8225$
2	$(2 - 3.35)^2 = 1.35^2 = 1.8225$
4	$(4 - 3.35)^2 = 0.65^2 = 0.4225$
4	$(4 - 3.35)^2 = 0.65^2 = 0.4225$
2	$(2 - 3.35)^2 = 1.35^2 = 1.8225$
4	$(4 - 3.35)^2 = 0.65^2 = 0.4225$
1	$(1 - 3.35)^2 = 2.35^2 = 5.5225$
1	$(1 - 3.35)^2 = 2.35^2 = 5.5225$
3	$(3 - 3.35)^2 = 0.35^2 = 0.1225$
4	$(4 - 3.35)^2 = 0.65^2 = 0.4225$
5	$(5 - 3.35)^2 = 1.65^2 = 2.7225$
3	$(3 - 3.35)^2 = 0.35^2 = 0.1225$
3	$(3 - 3.35)^2 = 0.35^2 = 0.1225$
4	$(4 - 3.35)^2 = 0.65^2 = 0.4225$
$\bar{x} = 3.35$	$s = \sqrt{\frac{34.55}{20}} = \sqrt{1.7275} \approx 1.31$

Step 3: The sum the numbers to the extreme right of the second column = 34.55

Step 4: Average the numbers in the second column: (34.55)/20 = 1.7275

Step 5: Take the square root of the number 1..7275:

$$s = \sqrt{\frac{34.55}{20}} = \sqrt{1.725} \approx 1.31$$

➤(d).
The variance is $s^2 = 1.725$.

3.2 - Computing the Variance and Standard Deviation for Frequency Distributions.

3.2 - Problem 1:
The following table computes the standard deviation and variance:

Weight Classes	(1) Mid-values	(2) frequency	(3) (1)x(2)	(4) $[(1)-\bar{x}]^2$	(5) (2)x(4)
[130,140)	135	15	2025	$(135 - 170.92)^2 = 1290.13$	15x1290.13 = 19351.94
[140,150)	145	45	6525	$(145 - 170.92)^2 = 671.76$	45x671.76 = 30229.28
[150,160)	155	55	8525	$(155 - 170.92)^2 = 253.39$	55x253.39 = 13936.69
[160,170)	165	42	6930	$(165 - 170.92)^2 = 35.03$	42x35.03 = 1471.14
[170,180)	175	32	6500	$(175 - 170.92)^2 = 16.66$	32x6500 = 533.11
[180,190)	185	48	8880	$(185 - 170.92)^2 = 198.29$	48x198.29 = 9518.03
[190,200)	195	25	4875	$(195 - 170.92)^2 = 579.93$	25x579.93 = 14498.13
[200,210)	205	12	2460	$(205 - 170.92)^2 = 1161.56$	12x1161.56 = 13938.69
[210,220)	215	9	1935	$(215 - 170.92)^2 = 1943.19$	9x1943.19 = 17488.71
[220,230)	225	9	2025	$(225 - 170.92)^2 = 2924.82$	9x2924.82 = 26323.41
[230,240)	235	2	470	$(235 - 170.92)^2 = 4106.46$	2x4106.91 = 8212.91
Total		294	50250		155502

Step 1: $\bar{x} = 50250/294 \approx 170.92$

Step 2: $s^2 = 155502/294 \approx 528.92$

Step 3: $s \approx \sqrt{528.92} \approx 23$

3.3 - An Application for the Standard Deviation

3.3 - Problem 1:

➤(a).
We complete the following table:

X	$(X - \bar{x})^2$
57.47	$(57.47 - 64.55)^2 = 50.1264$
60.05	$(60.05 - 64.55)^2 = 20.2500$
60.83	$(60.83 - 64.55)^2 = 13.8384$
61.78	$(61.78 - 64.55)^2 = 7.6729$
62.70	$(62.70 - 64.55)^2 = 3.4225$
62.73	$(62.73 - 64.55)^2 = 3.3124$
62.80	$(62.80 - 64.55)^2 = 3.0625$
63.16	$(63.16 - 64.55)^2 = 1.9321$
63.24	$(63.24 - 64.55)^2 = 1.7161$
63.27	$(63.27 - 64.55)^2 = 1.6384$
63.31	$(63.31 - 64.55)^2 = 1.5376$
63.94	$(63.94 - 64.55)^2 = 0.3721$
64.18	$(64.18 - 64.55)^2 = 0.1369$
64.68	$(64.68 - 64.55)^2 = 0.0169$
64.83	$(64.83 - 64.55)^2 = 0.0784$
64.97	$(64.97 - 64.55)^2 = 0.1764$
65.18	$(65.18 - 64.55)^2 = 0.3969$
65.26	$(65.26 - 64.55)^2 = 0.5041$
65.31	$(65.31 - 64.55)^2 = 0.5776$
65.51	$(65.51 - 64.55)^2 = 0.9216$
65.60	$(65.60 - 64.55)^2 = 1.1025$
65.88	$(65.88 - 64.55)^2 = 1.7689$
66.31	$(66.31 - 64.55)^2 = 3.0976$
66.46	$(66.46 - 64.55)^2 = 3.6481$
66.56	$(66.56 - 64.55)^2 = 4.0401$
67.21	$(67.21 - 64.55)^2 = 7.0756$
67.57	$(67.57 - 64.55)^2 = 9.1204$
67.64	$(67.64 - 64.55)^2 = 9.5481$
68.44	$(68.44 - 64.55)^2 = 15.1321$
69.61	$(69.61 - 64.55)^2 = 25.6036$
67.21	$(67.21 - 64.55)^2 = 7.0756$
67.57	$(67.57 - 64.55)^2 = 9.1204$
67.64	$(67.64 - 64.55)^2 = 9.5481$
68.44	$(68.44 - 64.55)^2 = 15.1321$
69.61	$(69.61 - 64.55)^2 = 25.6036$
Total = 1936.48	191.8272

$\bar{x} = 1936.48/30 \approx 64.55$

➤(b).

Step 1: $s^2 = 191.8272/30 = 6.39424$

Step 2: $s = \sqrt{6.39424} \approx 2.53$

➤(c).

Step 1: From the table we select all x values between $\bar{x} - s \leq x \leq \bar{x} + s$.

Step 2: $64.55 - 2.53 \leq x \leq 64.55 + 2.53$

Step 3: $62.02 \leq x \leq 67.08$

Step 4: The values of x from the table between $62.02 \leq x \leq 67.08$ are

62.70, 62.73, 62.80, 63.16, 63.24,
63.27, 63.31, 63.94, 64.18, 64.68,
64.83, 64.97, 65.18, 65.26, 65.31,
65.51, 65.60, 65.88, 66.31, 66.46,
66.56

➤(d).
Step 1: From the table we select all x values between $\bar{x} - 2s \leq x \leq \bar{x} + 2s$.

Step 2: $64.55 - 2(2.53) \leq x \leq 64.55 + 2(2.53)$

Step 3: $59.49 \leq x \leq 69.61$

Step 4: The values of x from the table between $59.49 \leq x \leq 69.61$ are

60.05, 60.83, 61.78, 62.70, 62.73,
62.80, 63.16, 63.24, 63.27, 63.31,
63.94, 64.18, 64.68, 64.83, 64.97,
65.18, 65.26, 65.31, 65.51, 65.60,
65.88, 66.31, 66.46, 66.56, 67.21,
67.57, 67.64, 68.44, 69.61

➤(e).
Step 1: From the table we select all x values between $\bar{x} - 3s \leq x \leq \bar{x} + 3s$.

Step 2: $64.55 - 3(2.53) \leq x \leq 64.55 + 3(2.53)$

Step 3: $56.96 \leq x \leq 72.14$

Step 4: The values of x from the table between $59.49 \leq x \leq 69.61$ are all the numbers.

➤(f).
From (d). we see that there are 29 numbers listed. This is equal to

$\frac{29}{30}\% \approx 96.67\%$ of all the numbers in the list.

Supplementary Problems

1.

➤ a.

Step 1:

$$1+ 2+ 3 +4 +...+ n = \frac{n(n + 1)}{2} = 0 + 1 + 2 + 3 + 4 + ... + 100$$

$$= \frac{100(100 + 1)}{2} = 5050.$$

Step 2: Since there are 101 numbers, $\bar{x} = 5050/101 = 50$.

➤ b.

Step 1: $(0 - 50)^2 + (1 - 50)^2 + (2 - 50)^2 + ... + (49 - 50)^2 + (50 - 50)^2 + + (51 - 50)^2 + (52 - 50)^2 +$

$... + (100 - 50)^2 =$

$$2[1^2 + 2^2 + 3^2 + 4^2 + ... + n^2] = 2\frac{n(n + 1)(2n + 1)}{6} =$$

$2[1^2 + 2^2 + 3^2 + ... + 50^2]$

$= 50^2 + 49^2 + 48^2 + ... + 1^2 + 0^2 + 1^2 + 2^2 + ... + 50^2 = 2(1^2 + 2^2 + ... + 50^2) =$

$$\frac{50(50 + 1)[2(50) + 1]}{3} = 85850.$$

Step 2: The variance $s^2 = 85850/101 = 850$.

Step 3: The standard deviation $s = \sqrt{850} \approx 29.15$.

➤ c.

The interval for 1 standard deviation of the mean is all numbers x where
$50 - 29.15 \le x \le 50 + 29.15$. Therefore, x = 21,22,23,....,79.

➤ d.

The interval for 2 standard deviations of the mean is all numbers x where
$50 - 2(29.15) \le x \le 50 + 2(29.15)$.
Therefore, x = 0,1,...,100.

2.

➤ a.

Complete the following table :

x	**$(x - \bar{x})^2$**
85	$(85 - 103.94)^2 = 358.7236$
85	$(85 - 103.94)^2 = 358.7236$
85	$(85 - 103.94)^2 = 358.7236$
86	$(86 - 103.94)^2 = 321.8436$
87	$(87 - 103.94)^2 = 286.9636$
89	$(89 - 103.94)^2 = 223.2036$
90	$(90 - 103.94)^2 = 194.3236$
91	$(91 - 103.94)^2 = 167.4436$
91	$(91 - 103.94)^2 = 167.4436$
92	$(92 - 103.94)^2 = 142.5636$
92	$(92 - 103.94)^2 = 142.5636$
92	$(92 - 103.94)^2 = 142.5636$
93	$(93 - 103.94)^2 = 119.6836$
93	$(93 - 103.94)^2 = 119.6836$
94	$(94 - 103.94)^2 = 98.8036$
97	$(97 - 103.94)^2 = 48.1636$
97	$(97 - 103.94)^2 = 48.1636$
99	$(99 - 103.94)^2 = 24.4036$
99	$(99 - 103.94)^2 = 24.4036$
100	$(100 - 103.94)^2 = 15.5236$
100	$(100 - 103.94)^2 = 15.5236$
103	$(103 - 103.94)^2 = 0.8836$
103	$(103 - 103.94)^2 = 0.8836$
103	$(103 - 103.94)^2 = 0.8836$
104	$(104 - 103.94)^2 = 0.0036$
106	$(106 - 103.94)^2 = 4.2436$
106	$(106 - 103.94)^2 = 4.2436$
106	$(106 - 103.94)^2 = 4.2436$
107	$(107 - 103.94)^2 = 9.3636$
108	$(108 - 103.94)^2 = 16.4836$
108	$(108 - 103.94)^2 = 16.4836$
108	$(108 - 103.94)^2 = 16.4836$
109	$(109 - 103.94)^2 = 25.6036$
109	$(109 - 103.94)^2 = 25.6036$
109	$(109 - 103.94)^2 = 25.6036$
112	$(112 - 103.94)^2 = 64.9636$
113	$(113 - 103.94)^2 = 82.0836$
114	$(114 - 103.94)^2 = 101.2036$
115	$(115 - 103.94)^2 = 122.3236$
116	$(116 - 103.94)^2 = 145.4436$
117	$(117 - 103.94)^2 = 170.5636$
117	$(117 - 103.94)^2 = 170.563$
118	$(118 - 103.94)^2 = 197.6836$
119	$(119 - 103.94)^2 = 226.8036$
Total = 5197	**Total = 6720.82**

Step 1: $\bar{x} = 5197/50 = 103.94$

Step 2: $s^2 = 6720.82/50 = 134.4164$

Step 3: $s = \sqrt{134.4164} \approx 11.59$.

➤ b.

Step 1: The following is a break-down of x:

$80 \le x < 90$: [80,90): 85, 85, 85, 86, 87, 89

$90 \le x < 100$: [90,100): 90, 91, 91, 92, 92, 92, 93, 93, 94, 97, 97, 99, 99

$100 \le x < 110$: [100,110): 100, 100, 103, 103, 103, 104, 106, 106, 106, 107, 108, 108, 108, 109, 109, 109

$110 \le x < 120$: [110,120): 112, 113, 114, 115, 116, 117, 117, 118, 119, 119

[120,130): 120, 121, 122, 123, 125

Step 2: This break-down gives the following frequency:

Classes	**Frequency**
[80,90)	6
[90,100)	13
[100,110)	16
[110,120)	10
[120,130)	5

➤ c.

We need to complete the following table:

Classes	**Mid-Value x**	**Frequency f**	$(x - \bar{x})^2$	$fx(x - \bar{x})^2$
[80,90)	85	6	$(85 - 104)^2$	$6(85 - 104)^2 = 2166$
[90,100)	95	13	$(95 - 104)^2$	$13(95 - 104)^2 = 1053$
[100,110)	105	16	$(105 - 104)^2$	$16(105 - 104)^2 = 16$
[110,120)	115	10	$(115 - 104)^2$	$10(115 - 104)^2 = 1210$
[120,130)	125	5	$(125 - 104)^2$	$5(125 - 104)^2 = 2205$
		Total = 50		6650

Step 1: $\bar{x} = \dfrac{85(6) + 95(13) + 105(16) + 115(10) + 125(5)}{50} = 104$

Step 2: $s^2 = 6650/50 = 133$

Step 3: $s = \sqrt{133} \approx 11.53$