Descriptive Statistics
Lesson 2
Averages

2.1 - How are the mean, median, and mode values computed?

2.1

➤(a).

Step 1: Sum the numbers 32.7, 31.3, 32.7, 29.7, 21.9, 31.3, 38.1, 37.7:

Sum = 32.7 + 31.3 + 32.7 + 29.7 + 21.9 + 31.3 + 38.1 + 37.7 = 255.40

Step 2: Divide sum = 255.40 by 8: $\overline{X}$ = Sum/8 = 31.925

➤(b).

The median of a set of ordered numbers is the middle value. Since there are eight values, we average the middle two values.

Step 1: We order the above numbers in ascending order:
21.9, 29.7, 31.3, 31.3, 32.7, 32.7, 37.7, 38.1

Step 2: The middle two numbers are 31.3 and 32.7.

Step 3: Average the two numbers in step 2: (31.3 + 32.7)/2 = 32.

Therefore, 32 is the median value.

➤(c).

The mode is the number(s) that most frequently occur in a set of data. In this set of data 31.3 and 32.7 occur twice while the other numbers only each occur once. Therefore, 31.3 and 32.7 are the modes.

2.1 Problem 2:

➤(a).

Step 1: Sum each row of the above numbers:

7 + 3 + 7 + 4 + 5 + 5 + 5 = 36
8 + 8 + 4 + 10 + 7 + 8 + 7 = 52

4 + 8 + 9 + 5 + 8 + 11+ 9 = 54
2 + 7 + 8 + 8 +6 + 8 + 8 = 47
5 + 4 + 7 + 9 +9 + 11 + 7 = 52

sum = 36 + 52 + 54 + 47 + 52 = 241

Step 2: Divide sum = 198 by 35: $\overline{X}$ = 241/35 ≈ 6.88.

➤(b).
The median of a set of ordered numbers is the middle value.

Step 1: We order the above numbers in ascending order:

2,3,4,4,4,4,5,5,5,5,5,6,7,7,7,7,7,7,7,8,8,8,8,8,8,8,8,8,9,9,9,9,10,11,11

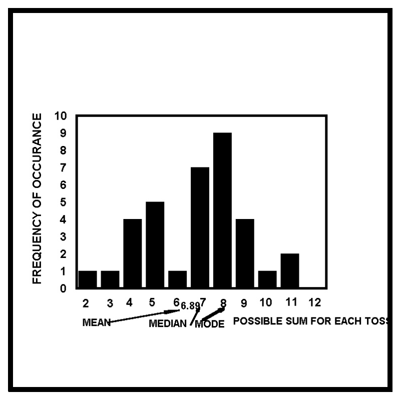
Step 2: Since there are an odd number of numbers (n = 35), we select the 18th number: 7.

➤(c).
The mode is the number(s) that most frequently occur in a set of data.
In this set of data the number 8 occurs 9 times. Therefore, 8 is the mode.

➤(d).
For this histogram, the horizontal axis contains all possible outcomes: 2,3,..., 12.

The vertical axis represents the frequency of occurrence of each of the possible outcomes. From the data the number 2 and 3 each occurred twice, 4 occurred four times, etc.

2.



2.2 - How are the mean, median, and modal values for a frequency distribution computed?

2.2- Problem 1:

➤ (a).
Step 1: The following table contains weight intervals and the mid-value for each interval:

Class	**MId-Values**	**Frequencies**	**Col 2 x Col 3**
[130,140)	135	15	135x15 = 2025
[140,150)	145	45	145x45 = 6525
[150,160)	155	55	155x55 = 8525
[160,170)	165	42	165x42 = 6930
[170,180)	175	32	175x32 = 5600
[180,190)	185	48	185x48 = 8880
[190,200)	195	25	195x25 = 4875
[200,210)	205	12	205x12 = 2460
[210,220)	215	9	215x9 = 1935
[220,230)	225	9	225x9 = 2025
[230,240)	235	2	235x2 = 470
		Total 294	**Total** 50250

Step 2: Mean = $\bar{x}$ = 50250/294 ≈ 170.92

➤(b).
Step 1: Divide the total by 2: 294/2 = 147

Step 2: Add up the frequencies until we get to 147:

0 + 15 = 15
15 + 45 = 60
15 + 45 + 55 = 115
15 + 45 + 55 + 42 = 157

Step 2: The number 147 is between 115 and 157.

Step 3: Therefore, the 157th member occurs in the class [160,170).

Step 4: The median value is given by the formula:

$$\frac{(147 - 115)}{147}(170 - 160) + 160 = 162.18 \text{ pounds}$$

➤(c).
Step 1: The class with the most frequent mode is [150,160).

Step 2: Let L = 150, the lowest value of the interval.

Step 3: Δ_1 = 55 - 45 = 10, the largest frequency value of the second column minus the preceding frequency value.

Step 4: Δ_2 = 55 - 42 = 13, the largest frequency value of the second column minus the following frequency value.

Step 5: Let C = 160 - 150 = 10.

Step 6: Using the formula: $L_1 + (\frac{\Delta_1}{\Delta_1 + \Delta_2})C = 150 + (\frac{10}{10 + 13})(10) \approx 154.35$

Supplementary Problems

1.

$$\frac{0.4524 + 0.4222 + 0.3987 + 0.4123 + 0.4454}{5} = \frac{2.131}{5} = 0.4262$$

$$0.4222 + 0.3987 + 0.4123 + 0.4454 + 0.5002 = \frac{2.1787}{5} = 0.4358$$

$$03987 + 0.4123 + 0.4454 + 0.5002 + 0.5112 = \frac{2.2678}{5} = 0.4536$$

$$0.4123 + 0.4454 + 0.5002 + 0.5112 + 0.5321 = \frac{2.4012}{5} = 0.4802$$

$$0.4454 + 0.5002 + 0.5112 + 0.5321 + 0.4986 = \frac{2.4875}{5} = 0.4975$$

$$0.5002 + 0.5112 + 0.5321 + 0.4986 + 0.4541 = \frac{2.4962}{5} = 0.4992$$

$$0.5112 + 0.5321 + 0.4986 + 0.4541 + 0.4199 = \frac{2.4159}{5} = 0.4832$$

$$0.5112 + 0.5321 + 0.4986 + 0.4541 + 0.4199 = \frac{2.4159}{5} = 0.4832$$

$$0.5321 + 0.4986 + 0.4541 + 0.4199 + 0.4041 = \frac{2.3088}{5} = 0.4618$$

$$0.4986 + 0.4541 + 0.4199 + 0.4041 + 0.3989 = \frac{2.1756}{5} = 0.4351$$

$$0.4541 + 0.4199 + 0.4041 + 0.3989 + 0.4000 = \frac{2.077}{5} = 0.4154$$

$$0.4199 + 0.4041 + 0.3989 + 0.4000 + 0.3910 = \frac{2.0139}{5} = 0.4028$$

$$0.4041 + 0.3989 + 0.4000 + 0.3910 + 0.3876 = \frac{1.9816}{5} = 0.3963$$

$$0.3989 + 0.4000 + 0.3910 + 0.3876 + 0.3818 = \frac{1.9593}{5} = 0.3919$$

$$0.4000 + 0.3910 + 0.3876 + 0.3818 + 0.3870 = \frac{1.9474}{5} = 0.3895$$

$$0.3910 + 0.3876 + 0.3818 + 0.3870 + 0.3987 = \frac{1.9461}{5} = 0.3892$$

$$0.3876 + 0.3818 + 0.3870 + 0.3987 + 0.3597 = \frac{1.915}{5} = 0.3830$$

2.
Step 1: Sum the ten numbers which includes the number p:

$4 + 12 + 654 + 132 + -10 + 13 + 0 + -125 + 13 + p = 693 + p.$

Step 2: Divide the sum in Step 1 by 10: $\frac{693 + p}{10}$.

Step 3: Set the expression in Step 2 to equal $\bar{x} = 1$: $\frac{693 + p}{10}$ **= 1.**

Step 4: Solve for p:

$693 + p = 1(10) = 10$

$p = 10 - 693 = -683.$

3.
Step 1: Write the sequence 5,6,7,8,9,...,10,000 as a sum:

5 + 6 + 7 + 8 + ...+ 10,000 .

Step 2: Write the sequence in Step 1 as

$5 + 6 + 7 + 8 + ... + 10{,}000 =$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + ... + 10{,}000 - (1 + 2 + 3 + 4)$.

Step 3: Matching the above formula with the sum in Step 1, we see that n = 10,000.

Step 4: Therefore, the formula can be used as follows:

$$5 + 6 + 7 + 8 + ... + 10{,}000 = \frac{(10000)(10001)}{2} - (1 + 2 + 3 + 4) = 50{,}004{,}990 - 10 = 50{,}004{,}990.$$

Step 5: Since there are 10,000 - 4 = 9,996 numbers in the sum, we divide the sum

5,004,990 by 9,996: $\bar{x} = \frac{50{,}004{,}990}{9{,}996} \approx 5002.50$.

4.

Step 1: Write the above sequence as a sum of squares:

$9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 + ... + 10{,}000 =$

$3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + ... + 100^2$.

Step 2: Write the above sequence as:

$3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + ... + 100^2 =$

$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + ... + 100^2 - 1^2 - 2^2$.

Step 3: Matching the above formula to the sum: n = 100.

Step 4: $1^2 + 2^2 + 3^2 + 4^2 + ... + 100^2 = \frac{100(100 + 1)(2(100) + 1)}{6} = 338{,}350.$

Step 5: $3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + ... + 100^2 =$

$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + ... + 100^2 - 1^2 - 2^2 =$

338,350 - 1 - 4 = 338,345

Step 6: Since there are 98 numbers in the sequence, $\bar{x} = \frac{338{,}345}{98} = 3{,}452.5.$

5.

Step 1: First we compute the average of the numbers 1,2,3,4,5,6,7,8,9,10:

$$\bar{x} = \frac{1+2+3+4+5+6+7+8+9+10}{10} = \frac{\frac{10(11)}{2}}{10} = \frac{55}{10} = 5.5.$$

Step 2: For each of the nine -number sequences, we compute their averages:

A1: $\bar{x} = \frac{2+3+4+5+6+7+8+9+10}{9} = \frac{55-1}{9} \approx 6$

A2: $\bar{x} = \frac{1+3+4+5+6+7+8+9+10}{9} = \frac{55-2}{9} \approx 5.89$

A3: $\bar{x} = \frac{1+2+4+5+6+7+8+9+10}{9} = \frac{55-3}{9} \approx 5.78$

A4: $\bar{x} = \frac{1+2+3+5+6+7+8+9+10}{9} = \frac{55-4}{9} \approx 5.67$

A5: $\bar{x} = \frac{1+2+3+4+6+7+8+9+10}{9} = \frac{55-5}{9} \approx 5.56$

A6: $\bar{x} = \frac{1+2+3+4+5+7+8+9+10}{9} = \frac{55-6}{9} \approx 5.44$

A7: $\bar{x} = \frac{1+2+3+4+5+6+8+9+10}{9} = \frac{55-7}{9} \approx 5.33$

A8: $\bar{x} = \frac{1+2+3+4+5+6+7+9+10}{9} = \frac{55-8}{9} \approx 5.22$

A9: $\bar{x} = \frac{1+2+3+4+5+6+7+8+10}{9} = \frac{55-9}{9} \approx 5.11$

A10: $\bar{x} = \frac{1+2+3+4+5+6+7+8+9}{9} = \frac{55-10}{9} = 5.$