

Descriptive Statistics

Lesson 1

Frequency Distributions

1.1 - What is a Frequency Distribution table and Histogram?

1.1 - Problem 1:

►(a).

# Possible Outcomes	Frequency of Occurrences	(b). Histogram
2	1	
3	1	
4	4	
5	5	
6	1	
7	7	
8	9	
9	4	
10	1	
11	2	
12	0	
Total	35	

Step 1: Scanning these values, we find the smallest value is 2 and the largest value is 11.

Step 2: From the data, we see that the number 2 occurred one time, 3 occurred one time, 4 occurred three times, 5 occurred 4 times, six occurred one time, 7 occurred seven times, 8 occurred nine times, 9 occurred four times, 10 occurred one time, 11 occurred two times, 12 did not occur ($12 = 6 + 6$ is a possible number).

➤(b).

# Possible Outcomes	Frequency of Occurrences	(b). Histogram
2	1	
3	1	
4	4	
5	5	
6	1	
7	7	
8	9	
9	4	
10	1	
11	2	
12	0	
Total	35	

1.1 - Problem 2:

➤(a).

Scanning the data, there are

- two runners that ran between 9.3 and 9.5 seconds,
- two runners ran between 9.5 and 9.7 seconds,
- one runner ran between 9.7 and 9.9 seconds,
- one runner ran between 9.9 and 10.1 seconds,
- two runners ran between 10.1 and 10.3 seconds,
- one runner ran between 10.3 and 10.5 seconds,
- three runners ran between 10.5 and 10.7 seconds,
- four runners ran between 10.7 and 10.9 seconds,
- one runner ran between 10.9 and 11.1 seconds,
- two runners ran between 11.1 and 11.3 seconds,
- one runners ran between 11.3 and 11.5 seconds.

►(b).

Racing speed in Seconds	Number of Racers	(b). Histogram
[9.3,9.5)	2	
[9.5,9.7)	2	
[9.7,9.9)	1	
[9.9,10.1)	1	
[10.1,10.3)	2	
[10.3,10.5)	1	
[10.5,10.7)	3	
[10.7,10.9)	4	
[10.9,11.1)	1	
[11.1,11.3)	2	
[11.3,11.5)	1	
Total	20	

1.2 - Relative-Frequency Distribution

1.2 - Problem 1:

►(a).

Step 1: The frequency distribution is

Hourly wage Classes	Number of Employees
[\$3.50,\$4.50)	4
[\$4.50,\$5.50)	14
[\$5.50,\$6.50)	16
[\$6.50,\$7.50)	12
[\$7.50,\$8.50)	3
[\$8.50,\$9.50)	1
Total	50

Step 2: In the above table, divide each number in the second column by 50:

$$4/50 = 0.08, 14/50 = 0.28, 16/50 = 0.32, 12/50 = 0.24, 3/50 = 0.06, 1/50 = 0.02$$

Hourly Wage Classes	Number of Employees
[\$3.5,\$4.5)	0.08
[\$4.5,\$5.5)	0.28
[\$5.5,\$6.5)	0.32
[\$6.5,\$7.5)	0.24
[\$7.5,\$8.5)	0.06
[\$8.5,\$9.5)	0.02
Total	1

►(b).

In the relative-frequency table, move each decimal number over two places to the right:

- 8% of the employees earn between \$3.50 and \$4.49 an hour.
- 28% of the employees earn between \$4.50 and \$5.49 an hour.
- 32% of the employees earn between \$5.50 and \$6.49 an hour.
- 24% of the employees earn between \$6.50 and \$7.49 an hour.
- 6% of the employees earn between \$7.50 and \$8.49 an hour.
- 2% of the employees earn between \$8.50 and \$9.49 an hour.

1.3 - Cumulative-Relative-Distribution

1.3 - Problem 1:

►(a).

Step 1: The relative-distribution table for this example is

Hourly Wage Classes	Number of Employees
[\$3.5,\$4.5)	0.08
[\$4.5,\$5.5)	0.28
[\$5.5,\$6.5)	0.32
[\$6.5,\$7.5)	0.24
[\$7.5,\$8.5)	0.06
[\$8.5,\$9.5)	0.02
Total	1

Step 2: For each line, sum the numbers at and above in the relative frequency table:

$$0.08 = 0.08$$

$$0.08 + 0.28 = 0.36$$

$$0.36 + 0.32 = 0.68$$

$$0.68 + 0.24 = 0.92$$

$$0.92 + 0.06 = 0.98$$

$$0.98 + 0.02 = 1.$$

Using these numbers we derive the cumulative distribution table:

Hourly Wage Classes	Number of Employees
[\$3.5,\$4.5)	0.08
[\$4.5,\$5.5)	0.36
[\$5.5,\$6.5)	0.68
[\$6.5,\$7.5)	0.92
[\$7.5,\$8.5)	0.98
[\$8.5,\$9.5)	1.00

►(b).

Eight percent earn less than \$4.50.

Thirty six percent earn less than \$5.50.

Sixty eight percent earn less than \$6.50.

Ninety two percent earn less than \$7.50.

Ninety eight percent earn less than \$8.50.

One hundred percent earn less than \$9.50.

Supplementary Problems

1.

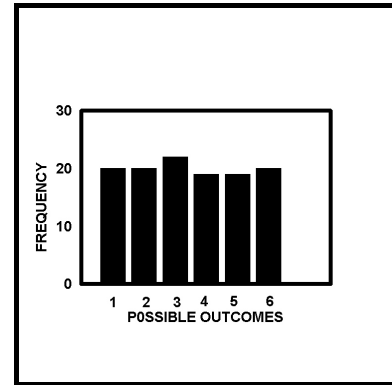
➤ a.

A die is tossed 120 times.

From the frequency distribution we know that:

- The number 1 occurred 20 times,
- The number 2 occurred 20 times,
- The number 3 occurred 22 times,
- The number 4 occurred 19 times,
- The number 5 occurred 19 times,
- The number 6 occurred 20 times.

1.



➤ b.

Since the six outcomes are discrete we construction the histogram as follows where the horizontal axis measures the six possible outcomes and the vertical axis measure the possible frequencies.

➤ c.

Each frequency value from the above table is used to compute the relative frequency outcome by dividing each of these numbers by 120:

$$20/120 \approx 0.17, 20/120 \approx 0.17, 22/120 \approx 0.18, 19/120 \approx 0.16, 19/120 \approx 0.16, 20/120 \approx 0.17$$

Single Class Outcomes	Relative Frequency for Each Outcome (rounded)
1	0.17
2	0.17
3	0.18
4	0.16
5	0.16
6	0.17
Total	1 (rounded)

➤ d.

We sum the values computed in the relative-frequency table to compute the cumulative relative frequency table:

Single Value Classes	Cumulative Relative Frequency for Each Outcome (rounded)
1	0.17
2	0.34
3	0.52
4	0.68
5	0.84
6	1.00

- 0.17 = 0.17
- 0.17 + 0.17 = 0.34
- 0.34 + 0.18 = 0.52
- 0.52 + 0.16 = 0.68
- 0.68 + 0.16 = 0.84
- 0.84 + 0.17 = 1.00 (rounded)

► e.
Out of 120 tosses of the die,

- Seventeen percent of the time the number 1 occurred.
- Thirty four percent of the time the numbers 1 or 2 occurred.
- Fifty two percent of the time the numbers 1,2,3 occurred.
- Sixty eight percent of the time the numbers 1,2,3,4 occurred.
- Eighty four percent of the time the numbers 1,2,3,4,5 occurred.
- One hundred percent of the time the numbers 1,2,3,4,5,6 occurred.

2.

- a.
- In each of twenty cities, ten children were surveyed as to their belief in Santa Claus.
- The number of cities where no children believe in Santa Claus is 0.
- The number of cities where exactly 1 child believes in Santa Claus is 1.
- The number of cities where exactly 2 children believe in Santa Claus is 2.
- The number of cities where exactly 3 children believe in Santa Claus is 2.
- The number of cities where exactly 4 children believe in Santa Claus is 6.
- The number of cities where exactly 5 children believe in Santa Claus is 4.
- The number of cities where exactly 6 children believe in Santa Claus is 2.
- The number of cities where exactly 7 children believe in Santa Claus is 2.
- The number of cities where exactly 8 children believe in Santa Claus is 1.
- The number of cities where exactly 9 children believe in Santa Claus is 0.
- The number of cities where exactly 10 children believe in Santa Claus is 0.

► b.

In the histogram, the horizontal axis represents the number of children that believe in Santa Clause, ranging from zero to ten.

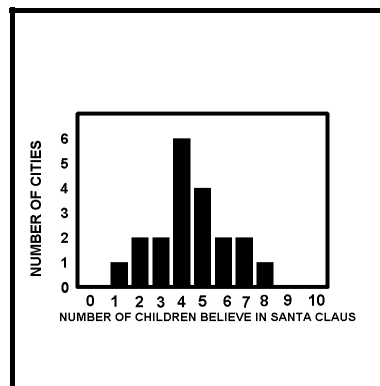
The vertical axis represents the number of cities for each number of children that believe in Santa Claus. For example, the vertical bar over 4 means that in six cities exactly 6 children out of 20 believed in Santa Clause.

► c.

We begin with the frequency distribution table above. Each entry in the right-hand column is divided by the total 20:

- 0/20 = 0
- 1/20 = 0.05
- 2/20 = 0.10
- 2/20 = 0.10
- 6/20 = 0.30
- 4/20 = 0.20
- 2/20 = 0.10
- 2/20 = 0.10
- 1/20 = 0.05
- 0/20 = 0
- 0/20 = 0

c.



We now use these numbers for the relative frequency distribution:

Number of Children that Believe in Santa Claus	Relative Frequency Distribution
0	0.00
1	0.05
2	0.10
3	0.10
4	0.30
5	0.20
6	0.10
7	0.10
8	0.05
9	0.00
10	0.00
Total	1

► d.

- In no city none of the children in the survey believe in Santa Claus.
- In 5% of the cities only one child in the survey believes in Santa Claus.
- In 10% of the cities only two children in the survey believe in Santa Claus.
- In 10% of the cities only three children in the survey believe in Santa Claus.
- In 30% of the cities only four children in the survey believe in Santa Claus.
- In 20% of the cities only five children in the survey believe in Santa Claus.
- In 10% of the cities only six children in the survey believe in Santa Claus.
- In 10% of the cities only seven children in the survey believe in Santa Claus.
- In 5% of the cities only eight children in the survey believe in Santa Claus.
- In no city did nine of the children in the survey believe in Santa Claus.
- In no city did ten of the children in the survey believe in Santa Claus.

3.

► a.

- All students have a grade point average of 4 or less.
- Ninety two percent of the students have a grade point average of 3.5 or less.
- Seventy eight percent of the students have a grade point average of 3.00 or less.
- Fifty six percent of the students have a grade point average of 2.50 or less.
- Forty one percent of the students have a grade point average of 2.00 or less.
- Eighteen percent of the students have a grade point average of 1.50 or less.
- Nine percent of the students have a grade point average of 1.00 or less.

► b.

To construct a relative frequency distribution from a cumulative relative frequency distribution we subtract, in the right column, the first line from the second, etc:

$$0.09 - 0 = 0.09, \quad 0.18 - 0.09 = 0.09, \quad 0.41 - 0.18 = 0.23, \quad 0.56 - 0.41 = 0.15, \quad 0.78 - 0.56 = 0.22$$

$$0.92 - 0.78 = 0.14, \quad 1.00 - 0.92 = 0.08,$$

These values are entered in the table:

Grade Point Average	Relative Frequency Distribution
[0.00,1.00)	0.09
[1.00,1.50)	0.09
[1.50,2.00)	0.23
[2.00,2.50)	0.15
[2.50,3.00)	0.22
[3.00,3.50)	0.14
[3.50,4.00]	0.08
Total	1

➤ c.

Nine percent of its students have a grade point average less than 1.00.

Nine percent of its students have a grade point average between 1.00 and 1.49.

Twenty three percent of its students have a grade point average between 1.50 and 1.99.

Fifteen percent of its students have a grade point average between 2.00 and 2.49.

Twenty two percent of its students have a grade point average between 2.50 and 2.99.

Fourteen percent of its students have a grade point average between 3.00 and 3.49.

Eight percent of its students have a grade point average between 3.50 and 4.00.

➤ d.

To compute a frequency distribution, we multiply each value in the right column of the relative frequency distribution by 10,000:

$$0.09 \times 10,000 = 900, \quad 0.09 \times 10,000 = 900, \quad 0.23 \times 10,000 = 2,300, \quad 0.15 \times 10,000 = 1,500$$

$$0.22 \times 10,000 = 2,200, \quad 0.14 \times 10,000 = 1,400, \quad 0.08 \times 10,000 = 800$$

These are the frequency values:

Grade Point Average	Frequency Distribution
[0.00,1.00)	900
[1.00,1.50)	900
[1.50,2.00)	2300
[2.00,2.50)	1500
[2.50,3.00)	2200
[3.00,3.50)	1400
[3.50,4.00]	800
Total	10,000